Wrocław University of Science and Technology
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Improving system operator electric demand predictions for quantile and density forecasting

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Motivation

Biased TSO forecasts

Simple linear regression

-37.9% in MAE

Probabilistic forecasting?

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Enhancing load, wind and solar generation for day-ahead forecasting of electricity prices

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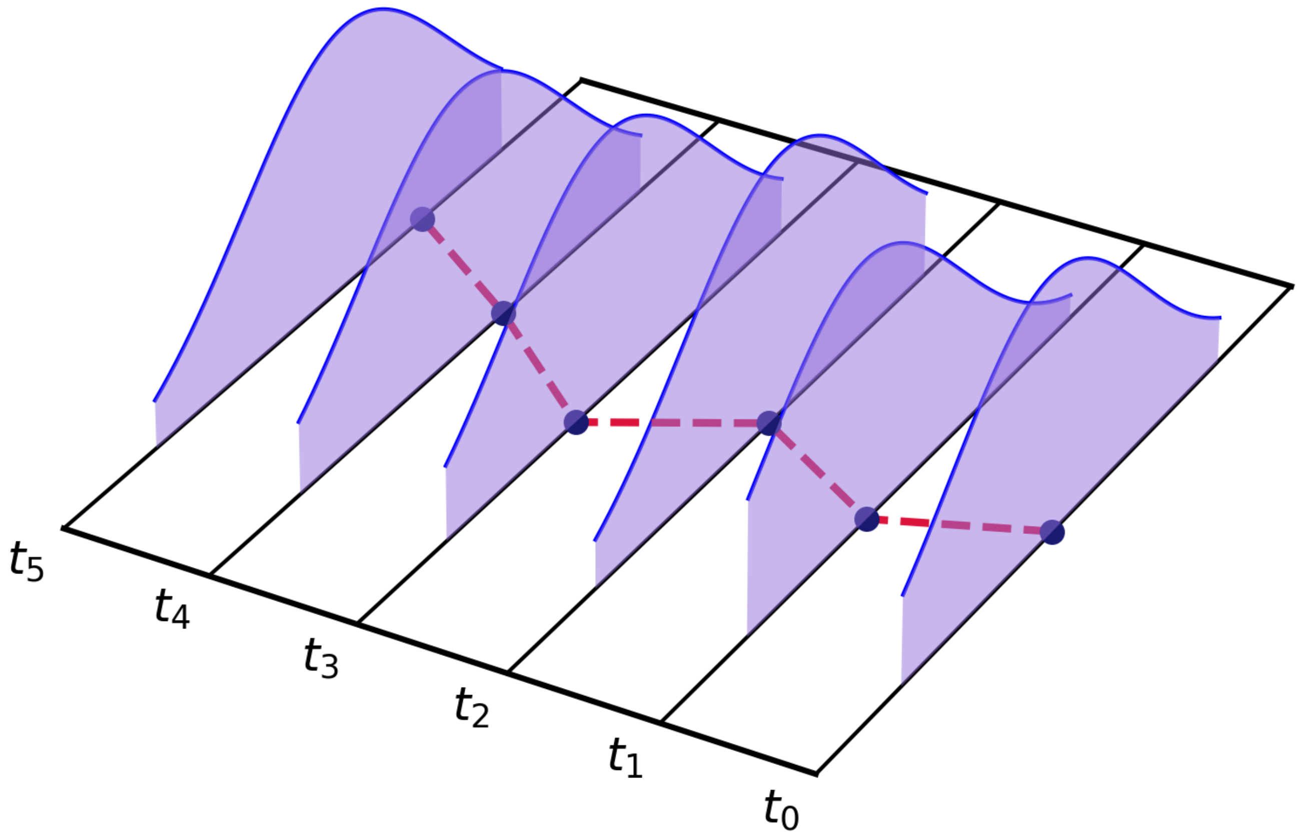
ABSTRACT

In recent years, a rapid development of renewable energy sources (RES) has been observed across the world. Intermittent energy sources, which depend strongly on weather conditions, induce additional uncertainty to the system and impact the level and variability of electricity prices. Predictions of RES, together with the level of demand, have been recognized as one of the most important determinants of future electricity prices. In this research, it is shown that forecasts of these fundamental variables, which are published by Transmission System Operators (TSO), are biased and could be improved with simple regression models. Enhanced predictions are next used for forecasting of spot and intraday prices in Germany. The results indicate that improving the forecasts of fundamentals leads to more accurate predictions of both, the spot and the intraday prices. Finally, it is demonstrated that utilization of enhanced forecasts is helpful in a day-ahead choice of a market (spot or intraday), and results in a substantial increase of revenues.

Probabilistic forecasting



Quantile forecasting

$\boldsymbol{Q_{0.01}, \dots, Q_{0.99}} \longrightarrow$ 

Density forecasting

$\boldsymbol{N(\mu, \sigma)} \longrightarrow$ 

GAMLSS framework

Chosen distribution

$$Y \sim D(\mu, \sigma, \nu, \tau)$$

Response variable

Distribution parameters

GAMLSS framework

$$Y \sim \mathcal{N}(\mu, \sigma^2)$$
$$\mu = X_1\beta_1$$
$$\sigma = X_2\beta_2$$

Classical linear regression

$$Y \sim \mathcal{N}(\mu, \sigma^2)$$
$$\mu = X_1\beta_1$$
$$\sigma = X_2\beta_2$$

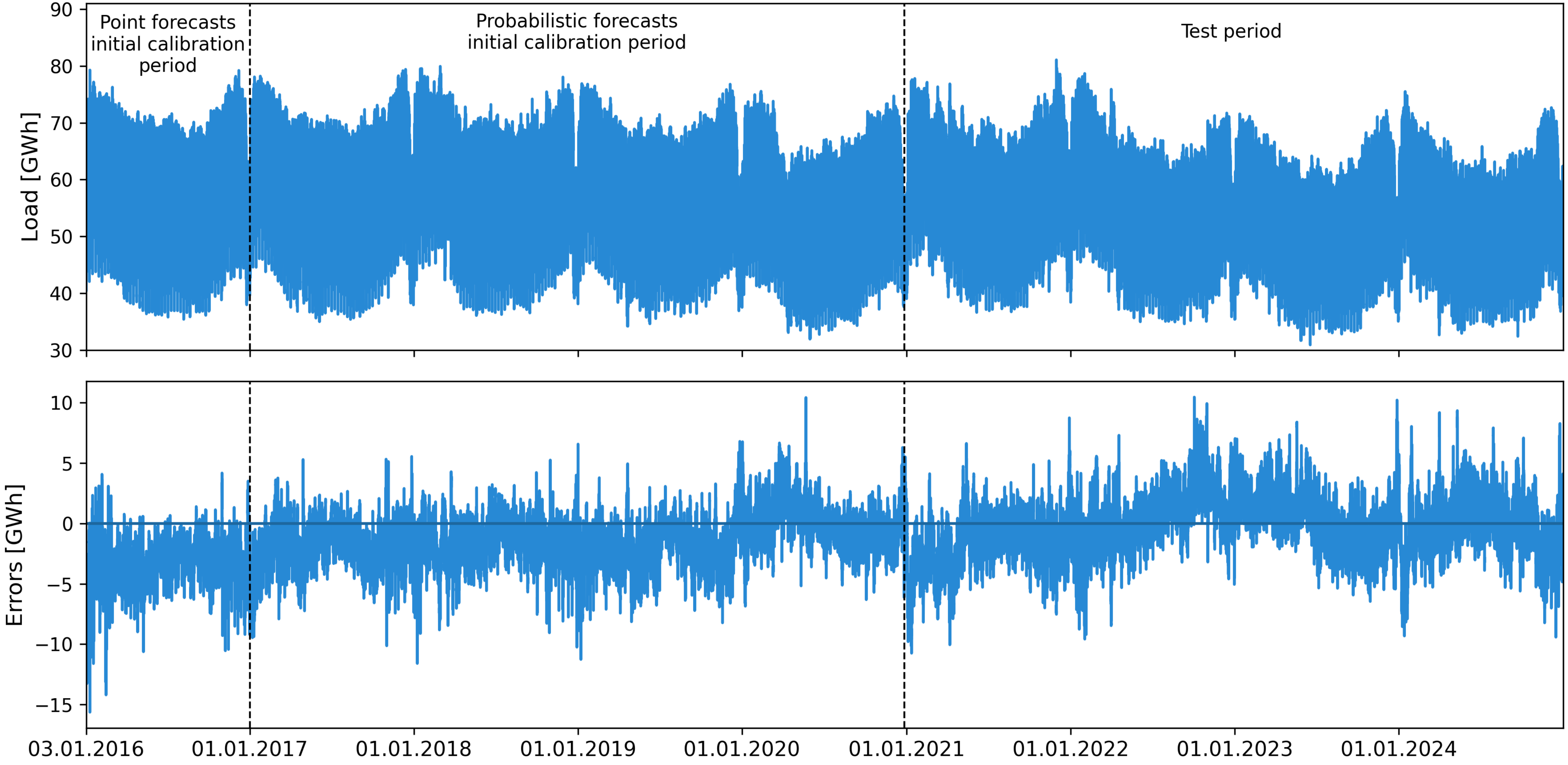
GAMLSS linear modeling

$$Y \sim \mathcal{N}(\mu, \sigma^2)$$
$$\mu = f_1(X_1)$$
$$\sigma = f_2(X_2)$$

GAMLSS non-linear modeling

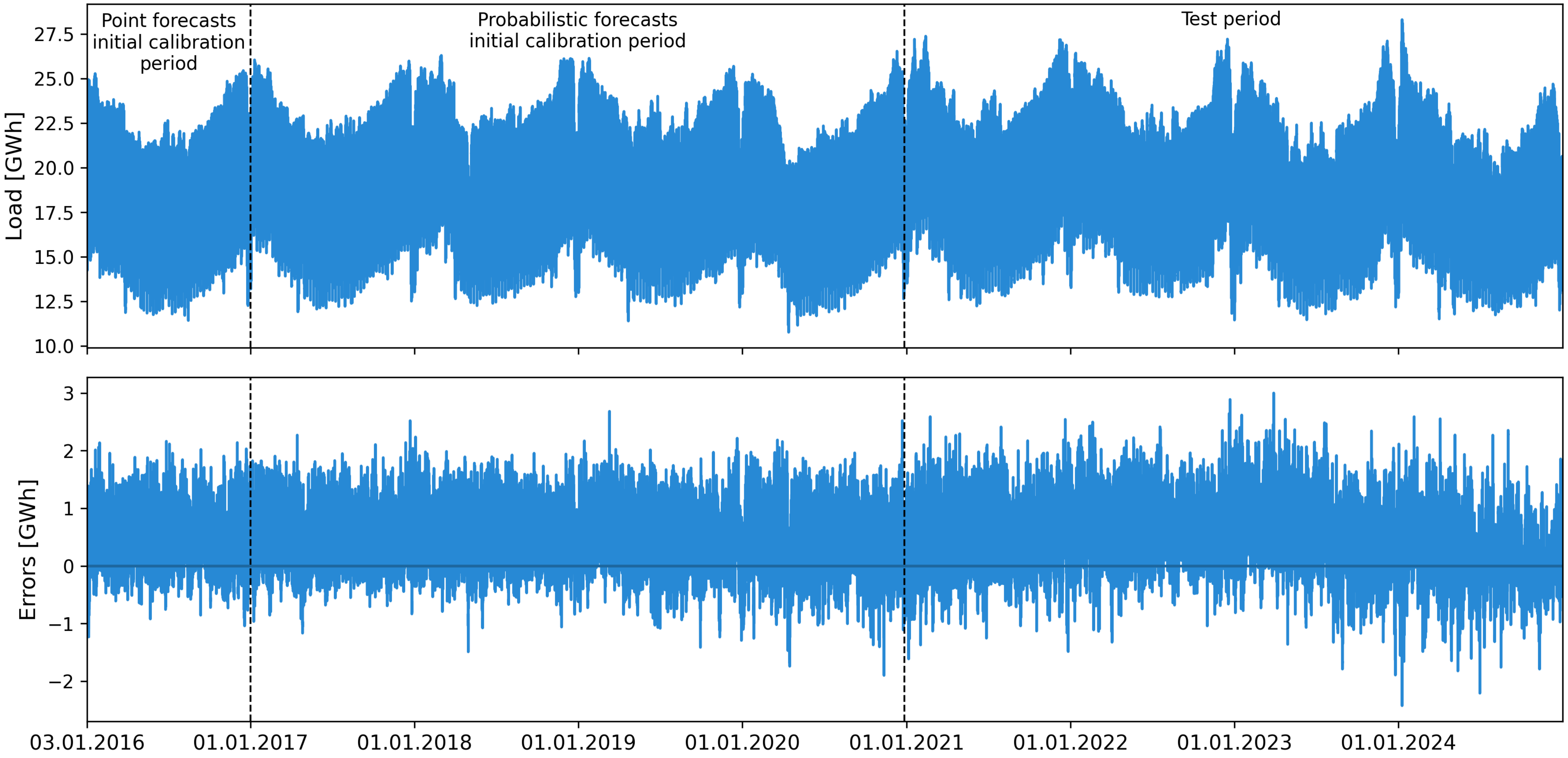
Rigby & Stasinopoulos (2005, J. R. Stat.)

German data

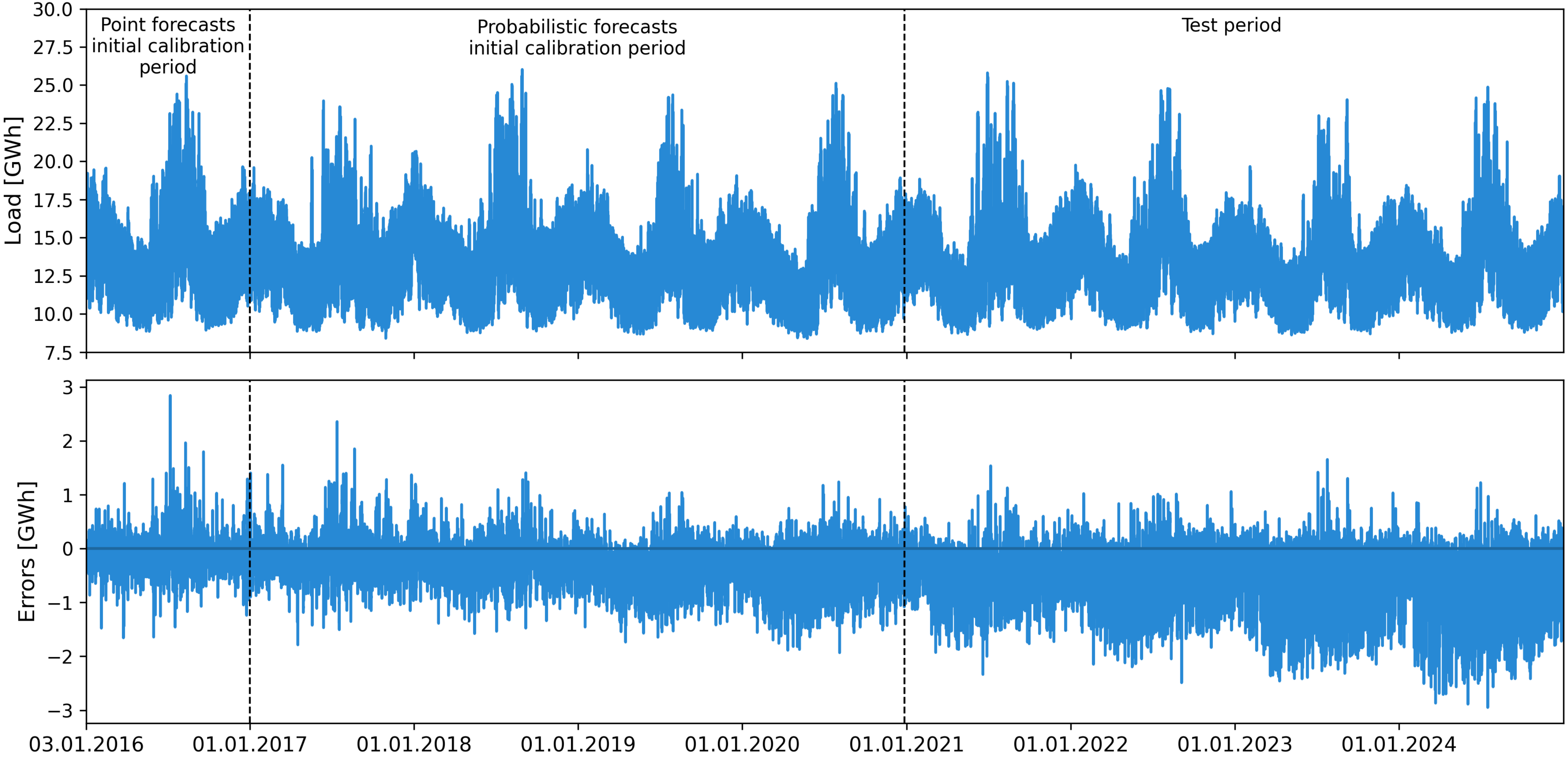


Point forecasts
initial calibration
period
Probabilistic forecasts
initial calibration period
Test period
Load [GWh]
90
80
70
60
50
40
30
Errors [GWh]
10
5
0
−5
−10
−15
03.01.2016
01.01.2017
01.01.2018
01.01.2019
01.01.2020
01.01.2021
01.01.2022
01.01.2023
01.01.2024

Polish data


Point forecasts
initial calibration
period
Probabilistic forecasts
initial calibration period
Test period
Load [GWh]
27.5
25.0
22.5
20.0
17.5
15.0
12.5
10.0
Errors [GWh]
3
2
1
0
−1
−2
03.01.2016
01.01.2017
01.01.2018
01.01.2019
01.01.2020
01.01.2021
01.01.2022
01.01.2023
01.01.2024

New England data



Methodology

Point forecasts

- TSO
- LASSO

Predictive distributions

- Quantile forecasting
- Density forecasting

Evaluation

- Pinball Score
- Diebold-Mariano test

Lasso Estimated AutoRegressive (LEAR) model

$$L_{d,h} = \underbrace{\sum_{i=1}^{24} \left(\beta_{h,i}\tilde{L}_{d,i} + \beta_{h,i+24}\tilde{L}_{d-1,i} + \beta_{h,i+48}\tilde{L}_{d-2,i} + \beta_{h,i+72}\tilde{L}_{d-7,i}\right)}_{\text{load forecasts effects}}$$

$$+ \underbrace{\sum_{i=1}^{24} \left(\beta_{h,i+96}L_{d-2,i} + \beta_{h,i+120}L_{d-7,i}\right)}_{\text{autoregressive effects}}$$

$$+ \underbrace{\sum_{i=1}^{24} \beta_{h,144+j}\hat{T}_{d,i}}_{\text{temperature forecasts}} + \underbrace{\sum_{j=1}^{7} \beta_{h,168+j}D_j}_{\text{daily dummies}} + \varepsilon_{d,h}$$

Lago et al. (2021, APEN), Wagner et al. (2022, JCM), Maciejowska et al. (2023, OUP)

GAMLSS model

$$g_k(\theta_{k_{d,h}}) = \beta_0 + \underbrace{\beta_1 \tilde{L}_{d,h} + \beta_2 \tilde{L}_{d-1,h} + \beta_3 \tilde{L}_{d-2,h} + \beta_4 \tilde{L}_{d-7,h}}_{\text{load forecasts effects}} + \underbrace{\beta_5 L_{d-2,h} + \beta_6 L_{d-7,h}}_{\text{autoregressive effects}} + \underbrace{\beta_7 f(\hat{T}_{d,h})}_{\text{temperature forecast}} + \underbrace{\sum_{j=1}^{6} \beta_{h,j+6} D_j}_{\text{daily dummies}} + \varepsilon_{d,h},$$

$$\tilde{L}_{d,h} = \hat{L}_{d,h}^{TSO} \vee \tilde{L}_{d,h} = \hat{L}_{d,h}^{LASSO}$$

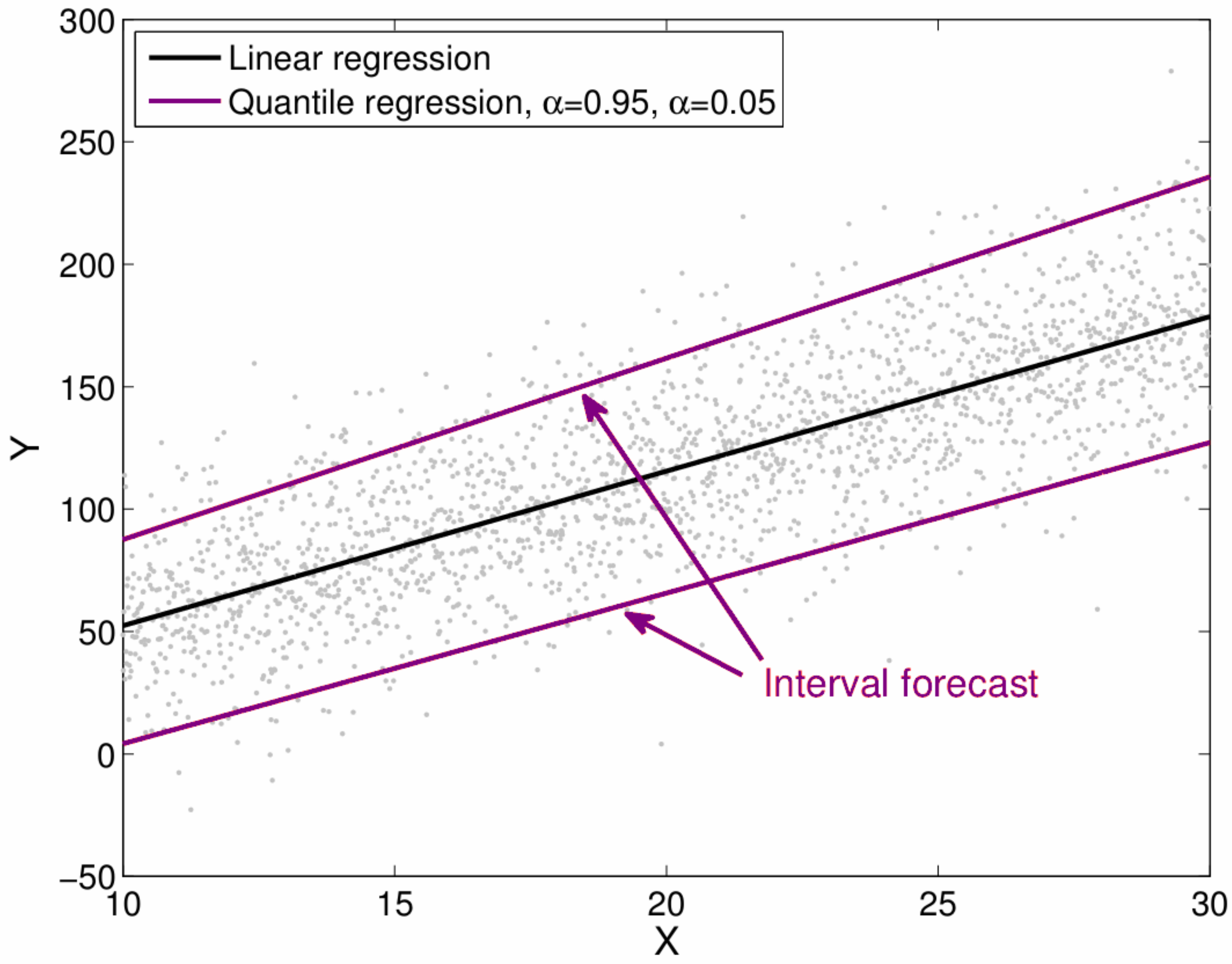
Misiorek et al. (2006, SNDE), Gaillard et al. (2016, IJF), Maciejowska et al. (2021, ENEECO), Taylor (2021, IJF), Maciejowska et al. (2023, OUP)

Quantile Regression (QR)

Involves solving a seperate minimization problem:

$$\min_{\boldsymbol{\beta}_q}\left[\sum_t \left(q - \mathbb{1}_{Y_t<\mathbf{X}_t\boldsymbol{\beta}_q}\right)\left(L_t - \mathbf{X}_t\boldsymbol{\beta}_q\right)\right]$$

for each quantile level (0.01,…,0.99)



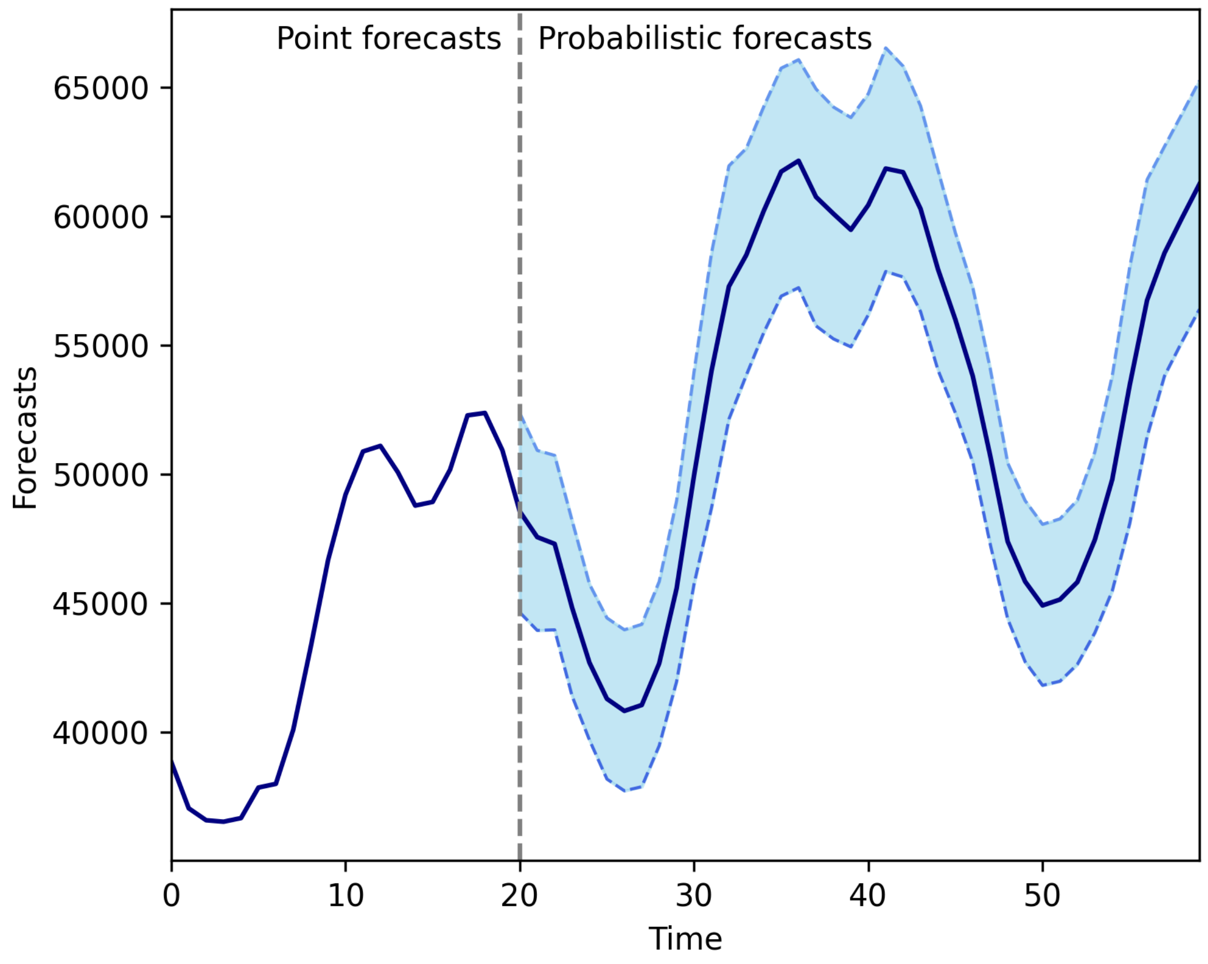
Postprocessing benchmarks

Historical Simulation (HS)

A **model-independent approach** that computes:

$$\hat{q}_{\tau|\hat{L}_t} = \hat{L}_t + Q_\tau(\varepsilon_t),$$

where $Q_\tau(\varepsilon_t)$ is the sample τ-quantile of errors $\varepsilon_t = L_t - \hat{L}_t$



For more details about postprocessing, see: Lipiecki et al. (2024), ENEECO

Pinball score

The **Pinball score** for quantile α:

$$PS\,(\hat{L}_t^{\alpha}, L_t, \alpha) = \begin{cases} (1-\alpha)(\hat{L}_t^{\alpha} - L_t) & \text{if } L_t < \hat{L}_t^{\alpha} \\ \alpha(L_t - \hat{L}_t^{\alpha}) & \text{if } L_t \geqslant \hat{L}_t^{\alpha} \end{cases}$$

where $\hat{L}_t^{\alpha}$ is the **α-th quantile** of the predictive distribution for time ***t***.

It can be averaged accross percentiles - **Aggregate Pinball Score (APS)**:

$$APS_{99} = \frac{1}{99} \sum_{\alpha=0.01}^{0.99} PS\,(\hat{L}_t^{\alpha}, L_t, \alpha)$$

Elliott & Timmermann (2016, PUP), Hong et al. (2016, IJF), Maciejowska et al. (2023, OUP)

Aggregate Pinball Score: 99 quantiles

	Quantile forecasting							Density forecasting					
								GAMLSS (N)		GAMLSS (T)		GAMLSS (JSU)	
	HS	QRA	CP	IDR	Avg	QR	$GAMLSS_0$	μ	μ, σ	μ	μ, σ	μ	μ, σ
Germany													
TSO	733.9	734.7	728.0	742.9	716.9	536.8	733.0	527.2	524.7	527.9	525.5	528.9	526.9
LASSO	499.1	498.7	497.6	509.2	497.5	492.9	499.1	482.6	479.1	482.5	479.2	483.3	480.4
Poland													
TSO	150.3	144.8	188.8	148.4	148.5	126.6	144.4	122.7	122.4	122.9	122.6	123.3	122.8
LASSO	121.6	121.7	121.6	126.2	121.8	119.2	121.5	115.2	115.3	115.4	115.5	115.7	115.7
New England													
TSO	91.4	84.9	157.5	81.8	96.5	73.4	86.0	62.0	60.6	61.2	60.1	61.2	60.2
LASSO	56.1	55.6	56.0	58.4	55.6	54.7	56.5	54.7	53.9	54.1	53.6	54.2	53.6

Results for test period: 27.12.2020 - 31.12.2024
For 208-weeks long rolling calibration window

Aggregate Pinball Score: 20 quantiles

	Quantile forecasting							Density forecasting					
	HS	QRA	CP	IDR	Avg	QR	$GAMLSS_0$	GAMLSS (N)		GAMLSS (T)		GAMLSS (JSU)	
								μ	μ, σ	μ	μ, σ	μ	μ, σ
Germany													
TSO	287.7	286.0	284.4	301.1	279.9	219.6	291.9	218.1	213.4	219.4	213.9	219.3	213.8
LASSO	204.7	204.8	204.5	218.1	204.8	201.2	204.6	199.2	193.9	200.2	193.9	200.5	194.1
Poland													
TSO	58.4	56.5	70.5	62.2	57.7	50.5	56.3	48.5	48.2	48.7	48.3	48.5	48.1
LASSO	47.9	48.0	47.8	53.5	48.3	47.7	47.7	45.5	45.4	45.6	45.5	45.7	45.5
New England													
TSO	35.6	32.9	55.4	36.1	37.5	29.4	34.5	26.0	24.7	25.7	24.6	25.8	24.7
LASSO	23.7	22.9	23.6	26.0	23.2	22.6	23.9	23.3	22.0	23.2	21.9	23.2	22.0

Results for test period: 27.12.2020 - 31.12.2024
For 208-weeks long rolling calibration window

Diebold-Mariano (DM) test

The error function:

$$\|\pi_d^A\|_1 = \sum_{h=1}^{24} |\pi_{d,h}^A|$$

The **'multivariate' loss differential series**:

$$\Delta_d^{A,B} = \|\pi_d^A\|_1 - \|\pi_d^B\|_1$$

The null hypothesis (H0):

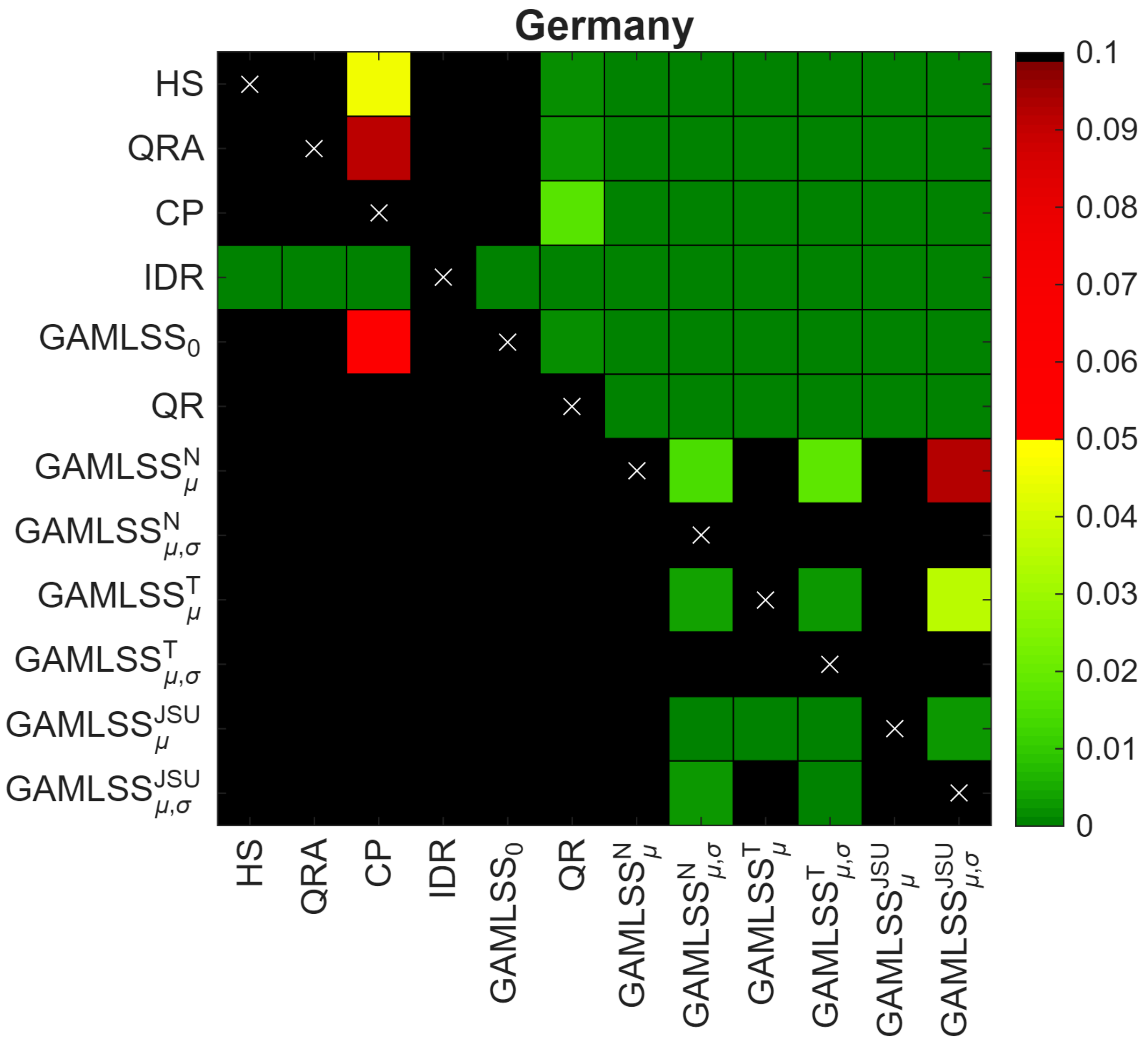
$$E(\Delta_d^{A,B}) \leq 0$$

The alternative hypothesis (H1):

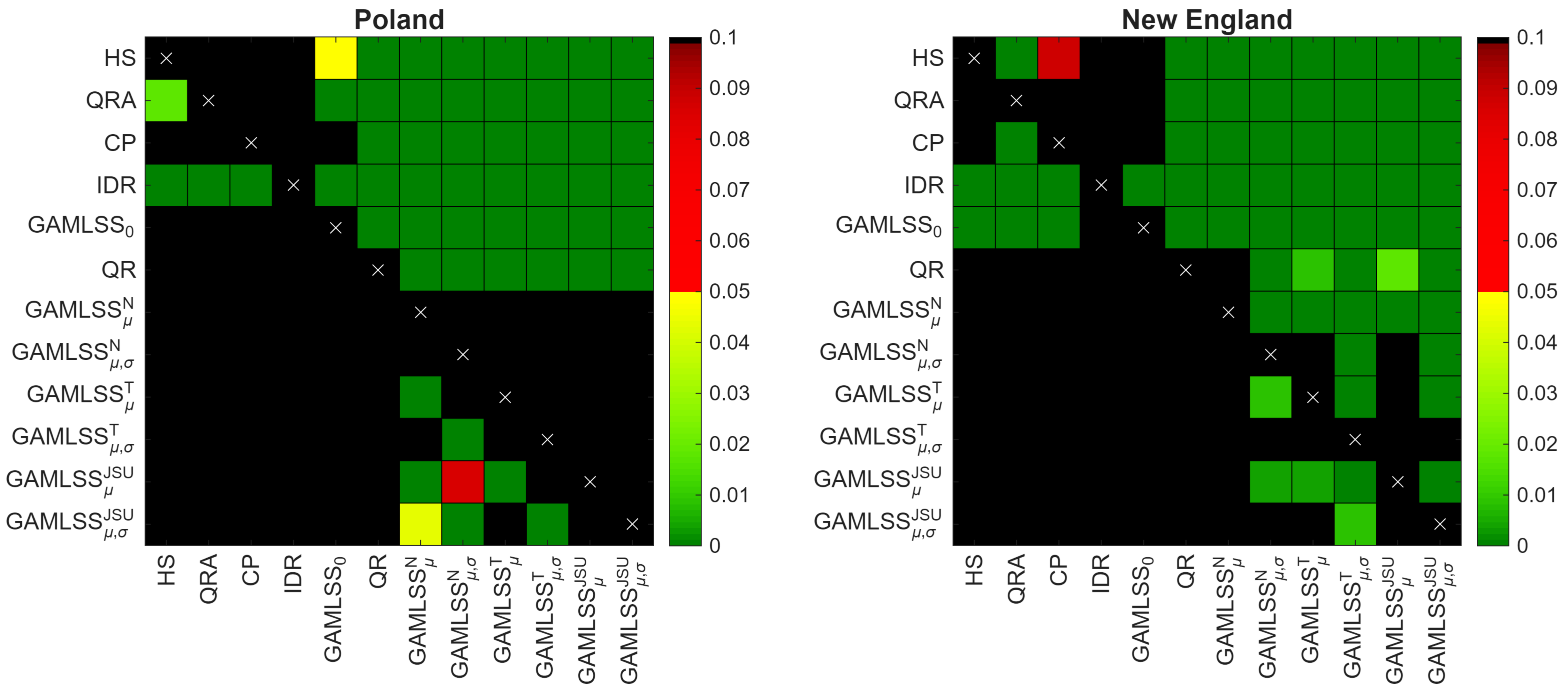
$$E(\Delta_d^{A,B}) \geq 0$$

Diebold & Mariano (1995, JBES), Ziel & Weron (2018, ENEECO), Lago et al. (2021, APEN)

Diebold-Mariano (DM) test



Diebold-Mariano (DM) test



Conclusions

GAMLSS is able to outperform the benchmarks while remaining computationally efficient

Usage of LASSO point forecast instead of the TSO significantly improves the results

The observed improvement is confirmed on 3 different markets with DM test

Thank you for attention!

Postprocessing benchmarks

Conformal Prediction (CP)

The αth quantile is given by:

$$\hat{q}(\alpha|\hat{L}_t) = \begin{cases} \hat{L}_t - \lambda^{2\alpha} & \text{if } \alpha < 0.5, \\ \hat{L}_t + \lambda^{2(1-\alpha)} & \text{otherwise} \end{cases}$$

where λ^{α} is the so-called **nonconformity score**

Historical Simulation (HS)

A **model-independent approach** that computes:

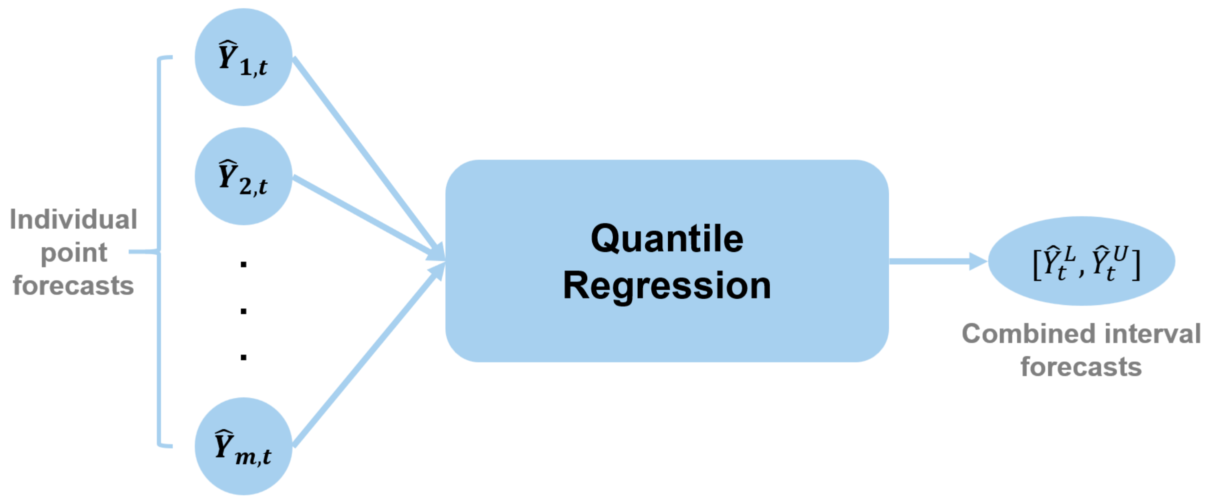
$$\hat{q}_{\tau|\hat{y}_t} = \hat{y}_t + Q_\tau(\varepsilon_t),$$

where $Q_\tau(\varepsilon_t)$ is the sample τ-quantile of errors $\varepsilon_t = y_t - \hat{y}_t$

For more details, see: Lipiecki et al. (2024), ENEECO

Postprocessing benchmarks

Quantile Regression Averaging (QRA)



Isotonic Distributional Regression (IDR)

- A **nonparametric** method
- The output $\widehat{F}$ minimizes the CRPS
- **Isotonic constraint**: the quantiles of the response must be non-decreasing with respect to the regressor

For more details, see: Lipiecki et al. (2024), ENEECO

Diebold-Mariano (DM) test