

# Trend and long-run relations in electricity prices

## Why pre-filtering is inevitable

Matteo Pelagatti<sup>2</sup>

jointly with

A. Gianfreda<sup>1</sup> L. Parisio<sup>2</sup> P. Maranzano<sup>2</sup>

<sup>1</sup>Free University of Bozen

<sup>2</sup>University of Milano - Bicocca

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# Motivation

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## [HTML] Regime jumps in **electricity** prices

[R Huisman](#), R Mahieu - *Energy economics*, 2003 - Elsevier

... This highlights the importance of **mean-reversion** in **electricity price** processes ... Standard errors are between parentheses. Note that the inclusion of **mean-reversion** leads to a richer specification of the **electricity price** process indicated by the lower log-likelihood values ...

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## [BOOK] Stochastic models of energy commodity prices and their applications: **Mean-reversion** with jumps and spikes

[S Deng](#) - 2000 - [ei.haas.berkeley.edu](http://ei.haas.berkeley.edu)

... congestion on key transmission lines. Within a couple of days the **price** fell back to the ... Figure 3: Generation Stack for **Electricity** in a Region Although the **mean reversion** is well studied, there has been little work examining the ...

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# Motivation

- Many authors list **mean reversion** as an important feature of electricity prices.
- In most electricity markets **gas, coal and oil prices** are **important drivers** of electricity prices.
- There is virtual unanimity in holding **gas, coal and oil log-price dynamics** as well approximated by **integrated processes** (mostly as random walks).
- Therefore, mean-reversion of electricity prices does not seem plausible for most markets.

## Why do researchers find/assume mean reversion?

I take da Silva, Soares (2007) as one of many examples.

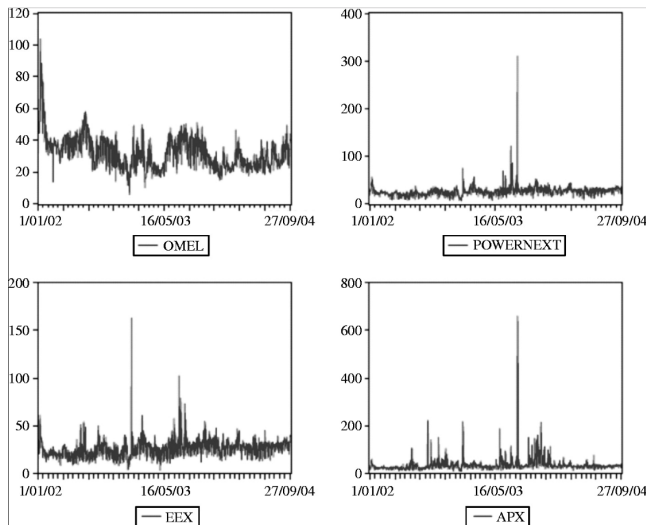
*According to both unit root tests, the hypothesis of stationarity cannot be rejected for the four series. Hence, although very appealing, co-integration does not seem the appropriate method to use in the analysis; as such an analysis would always find prices to be co-integrated.*

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	OMEL	POWERNEXT	APX	EEX
ADF	-5.401	-10.540	-10.512	-10.560
PP	-9.760	-22.585	-17.639	-18.183

**Notes:** 1 per cent Critical value (MacKinnon critical values for rejection of hypothesis of a unit root), -3.4396; 5 per cent Critical value, -2.8648; 10 per cent Critical value, -2.5685

# Can you use least-squares based methods on these data?



# The drivers of electricity prices

- Electricity prices are determined in the long-run mainly by demand level, fuel prices and the mix of generation technologies.
- Hourly and daily prices are buried into high-variance leptokurtic noise produced by many factors:
  - ▶ line congestions (Italy can be split in up to six zones if this happens),
  - ▶ firms' strategies and, possibly, exercise of market power at particular hours/seasons,
  - ▶ plant maintenance (both programmed and unexpected),
  - ▶ start-up costs of power plants,
  - ▶ intermittent RES generation,
  - ▶ import/export prices effects.

## How important is the noise in daily prices?

Consider the Unobserved Component Model

$$\log(p_t) = \mu_t + \gamma_t + \omega_t + \beta^\top \mathbf{x}_t + \varepsilon_t,$$

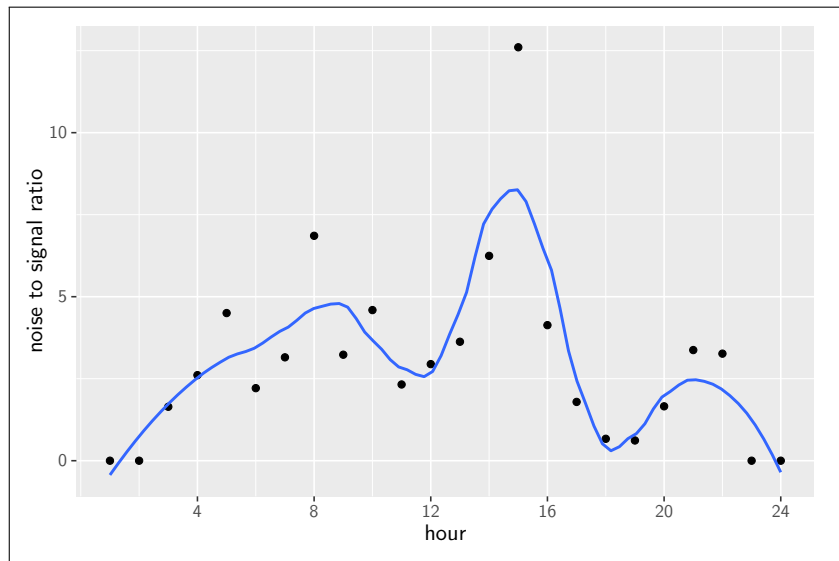
with

- $p_t$  are daily prices of a given hour,
- $\mu_t = \mu_{t-1} + \eta_t$  is a random walk trend component,
- $\gamma_t$  is a stochastic dummy weakly seasonal component,
- $\omega_t$  is a smooth yearly seasonal component made of stochastic sinusoids,
- $\beta^\top \mathbf{x}_t$  are regression effects capturing holidays,
- $\varepsilon_t$  is the noise component;

and where the noise-to-signal ratio is defined as

$$c = \frac{\text{VAR}(\varepsilon_t)}{\text{VAR}(\eta_t)}.$$

## Noise-to-signal ratio in Italian electricity prices





## I(1) process observed with noise

Let us consider the simplest I(1) process, the random walk, observed with noise

$$\begin{aligned}y_t &= x_t + \varepsilon_t, & \varepsilon_t &\sim \text{WN}(\sigma_\varepsilon^2) \\x_t &= x_{t-1} + \eta_t, & \eta_t &\sim \text{WN}(\sigma_\eta^2)\end{aligned}$$

This process has the reduced ARIMA(0, 1, 1) form

$$\Delta y_t = \eta_t + \varepsilon_t - \varepsilon_{t-1} = \zeta_t - \theta \zeta_{t-1}, \quad \zeta_t \sim \text{WN}(\sigma^2)$$

with

$$\theta = 1 + \frac{\lambda - \sqrt{\lambda^2 + 4\lambda}}{2}, \quad \sigma^2 = \frac{\sigma_\varepsilon^2}{\theta}.$$

where  $\lambda = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$  is the signal-to-noise ratio.

## I(1) process observed with noise

If the signal-to-noise ratio is low, then the coefficient  $\theta$  is close to one and the MA operator almost annihilate the difference operator:

$$(1 - L)y_t = (1 - \theta L)\zeta_t,$$

and the I(1) process is hard to distinguish from a white noise.

At the same time, if, as in ADF test, the MA(1) has to be approximated by an AR( $p$ ), when  $\theta$  is close to one, the value of  $p$  must be very large to obtain a decent approximation:

$$(1 - \theta L)^{-1} = 1 + \theta L + \theta^2 L^2 + \theta^3 L^3 + \dots$$

Thus, the critical values of the ADF will be bad approximations as well.

The closer  $\theta$  to zero, the better for ADF and Johansen tests.

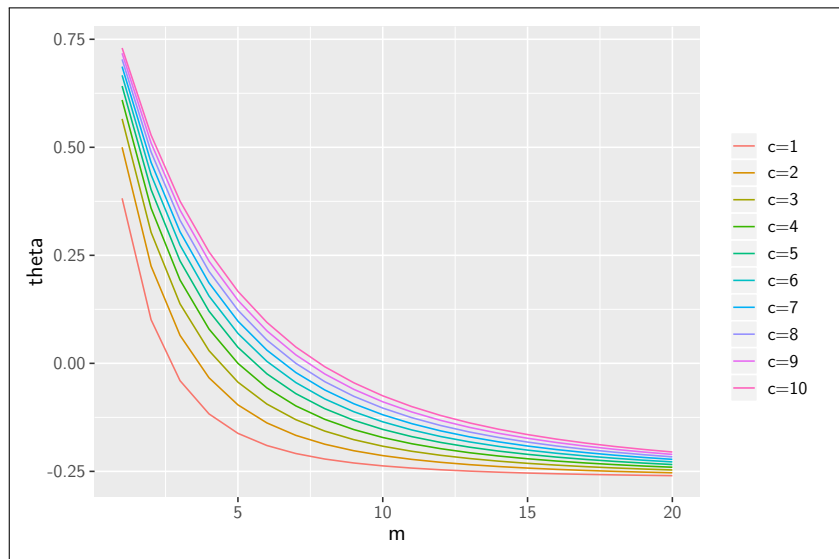
## How we dealt with the problem

To study integration and RES effects on some European electricity markets we filtered price time series using one or more of the following approaches.

- Taking **weekly means or medians** (Bosco et al. 2010, Gianfreda et al. 2016).
- Developing **robust integration and cointegration tests** (Bosco et al. 2010, Pelagatti & Sen 2013).
- Extracting the long-run component using UCM and **Kalman filtering/smoothing** (Gianfreda et al. 2016, 2018).

In this talk, we want to show the advantages of using these filtering methods when working on electricity prices.

The mean over  $m$ -observations of a RW + WN is IMA(1, 1) with the following  $\theta$



# Data generating process for the ADF test

The data are generated by summing a random walk

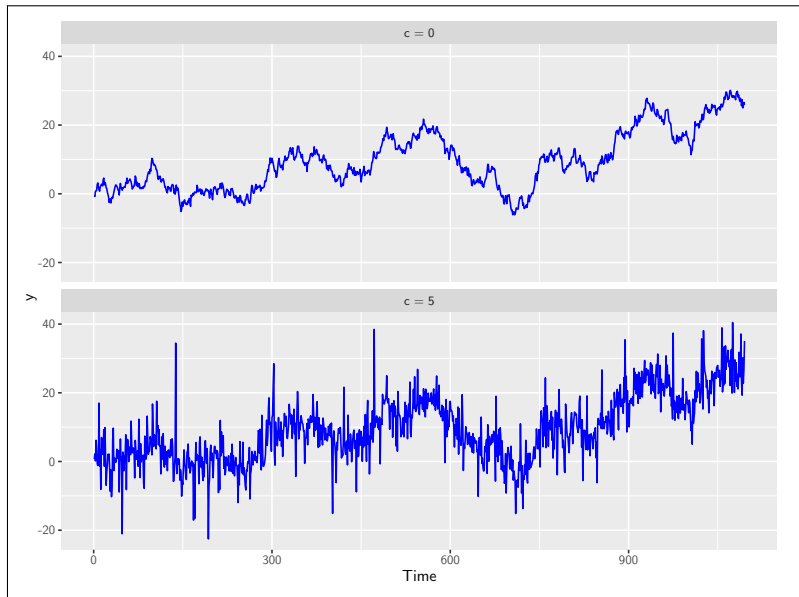
$$x_t = x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.N(0, 1),$$

with a **leptokurtic noise**  $z_t$ , generated by a (standardized) Student's  $t$  random variable with  $\nu$  degrees of freedom:

$$y_t = x_t + \sqrt{c}z_t.$$

The parameter  $c$  is fixed and represents the **noise-to-signal ratio**.

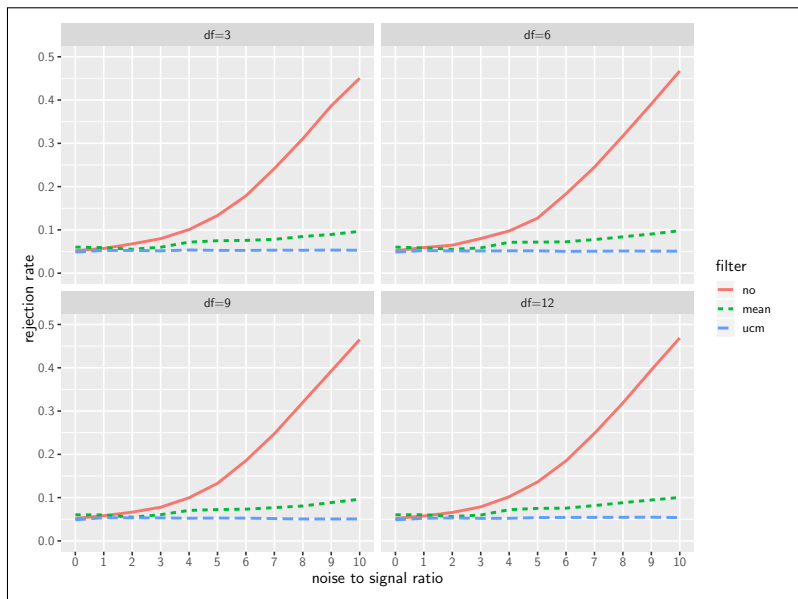
# Sample path of RW and RW + NOISE



## Simulation scheme

- We simulated 10,000 time series paths of length 1095 (3 years of daily observations) for the following values:
  - noise-to-signal ratio  $c = 0, 1, \dots, 10$ ;
  - Student's  $t$  degrees of freedom  $df = 3, 6, 9, 12$ .
- On each simulated time series we applied the ADF test with the number of lags determined using AIC (max lags = 10).
- We applied the ADF test to:
  - no raw data
  - mean aggregated time series by taking the mean every 7 observations (from daily to weekly)
  - ucm smoothed level in a *random walk plus noise* UCM (variances estimated by Gaussian QML)

# Rejection rate of the ADF test





## Data generating processes for Johansen's test

We assume that our  $k = 4$  time series in  $\mathbf{y}_t$  are generated by the following VECMs with  $r = 1$  and  $r = 2$  cointegrating relations

$$\Delta \mathbf{x}_t = \begin{bmatrix} 0.0 & 0.0 \\ 0.1 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.2 \end{bmatrix} \begin{bmatrix} 1.0 & -0.5 & -0.5 & 0.0 \\ 0.0 & 1.0 & 0.0 & -1.0 \end{bmatrix} \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,$$

$$\Delta \mathbf{x}_t = \begin{bmatrix} 0.0 \\ 0.1 \\ -0.1 \\ 0.1 \end{bmatrix} [1 \quad -1 \quad 1 \quad -1] \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,$$

plus a leptokurtic noise  $z_{it}$ , generated by (standardized) Student's  $t$  random variables with  $\nu$  degrees of freedom:

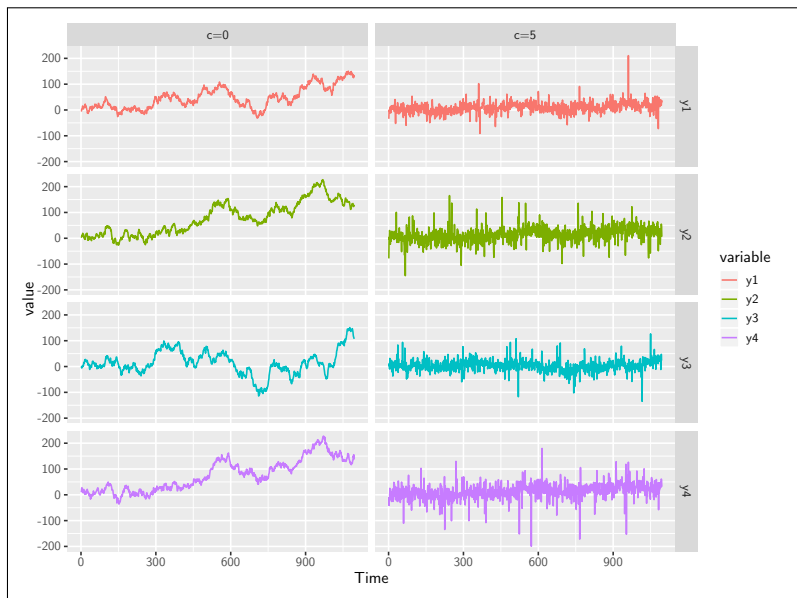
$$y_{it} = x_{it} + \gamma_i z_{it}, \quad \gamma_i^2 = c \frac{\text{VAR}(\Delta y_{it})}{\text{VAR}(z_{it})}.$$

The parameter  $c$  is fixed and represents the noise-to-signal ratio.

# Simulation scheme

- The characteristic roots of the above model written in VAR(1) form are, respectively: 1.0 1.0 1.0 0.7 and 1.00 1.00 0.85 0.80
- We simulated 10,000 time series paths of length 1095 (3 years of daily observations) for the following values:
  - noise-to-signal ratio  $c = 0, 1, 2, \dots, 10$
  - Student's  $t$  degrees of freedom  $df = 3, 6, 9, 12$
- On each simulated time series quartet, we applied Johansen's trace test.

# Sample path of VECM and VECM + NOISE

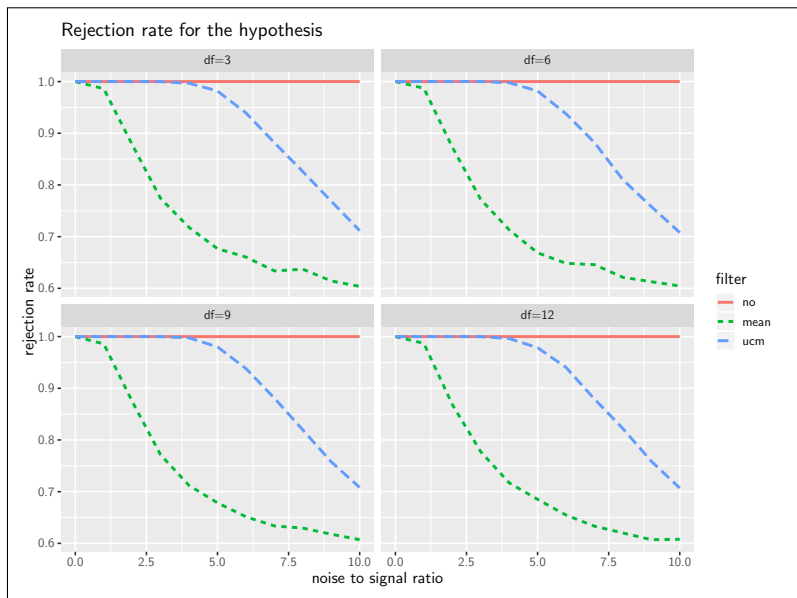


# The filters we used

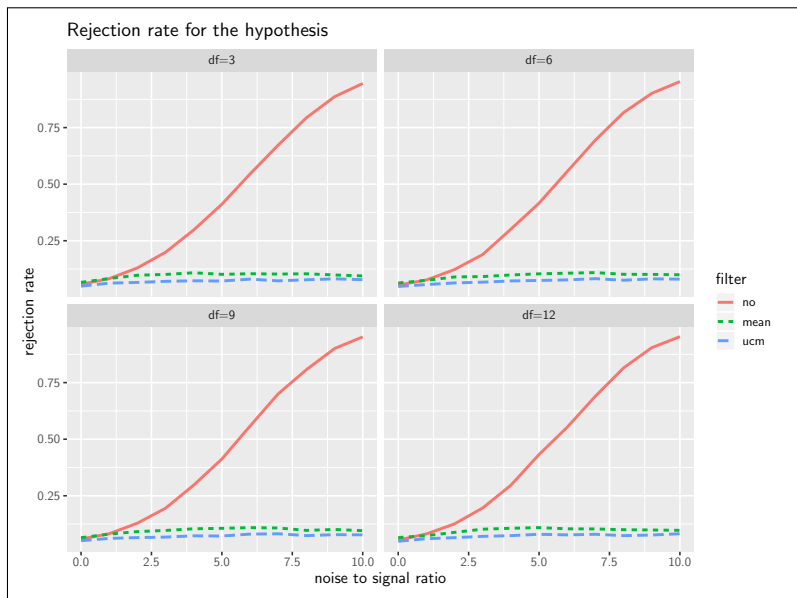
We applied the Johansen test to:

- no** raw data
- mean** aggregated time series by taking the mean every 7 observations (from daily to weekly)
- ucm** smoothed level in a *random walk plus noise* UCM (variances estimated by Gaussian QML)

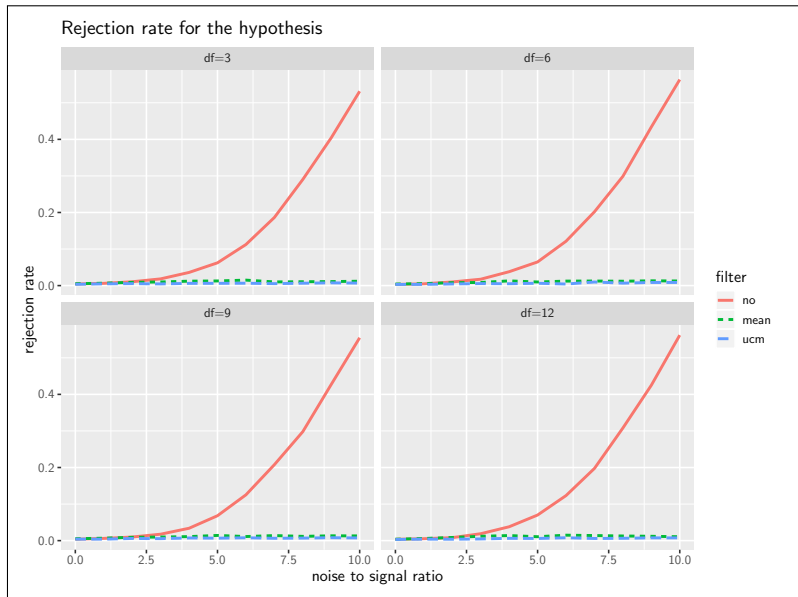
# Rejection rates for $H_0 : r = 0$ when $r = 1$



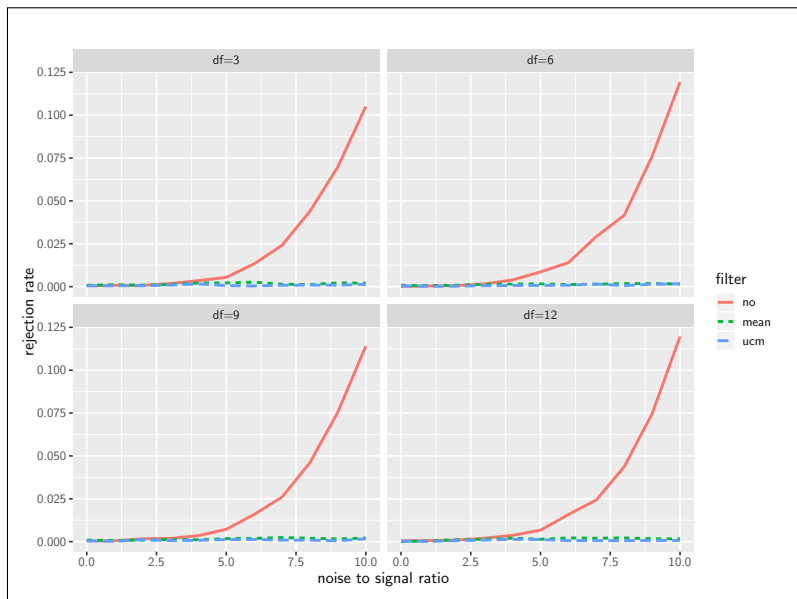
# Rejection rates for $H_0 : r = 1$ when $r = 1$



# Rejection rates for $H_0 : r = 2$ when $r = 1$

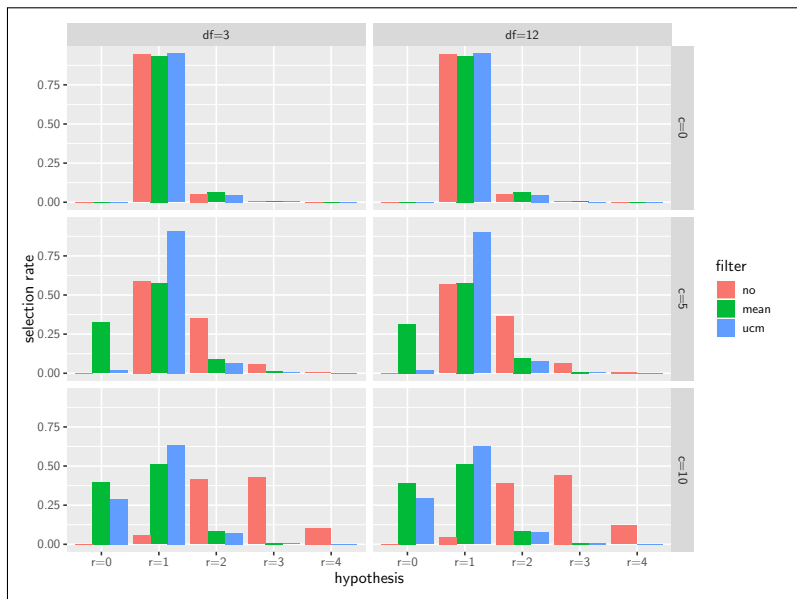


# Rejection rates for $H_0 : r = 3$ when $r = 1$

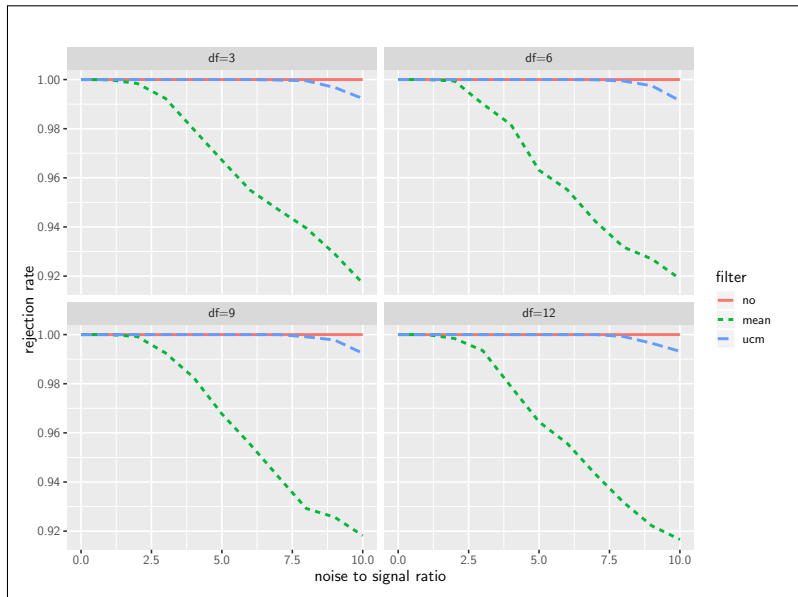




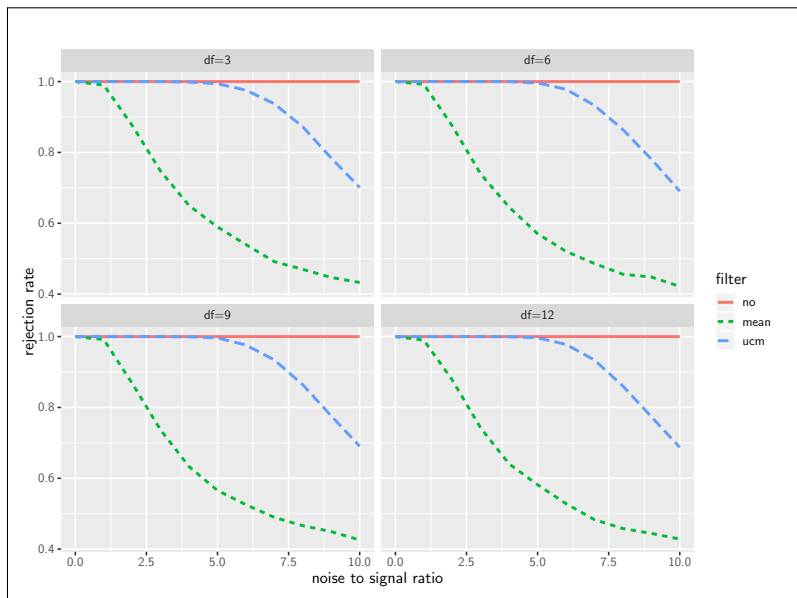
# Selection rates when $r = 1$



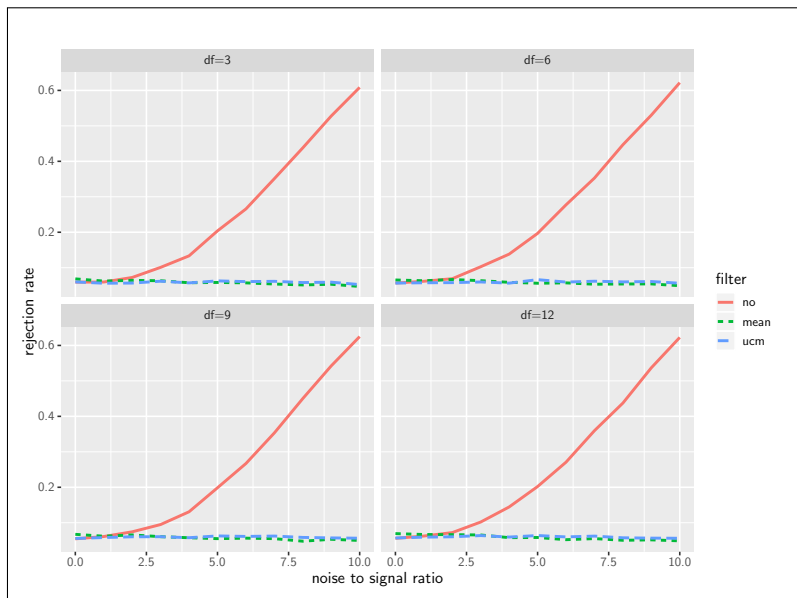
# Rejection rates for $H_0 : r = 0$ when $r = 2$



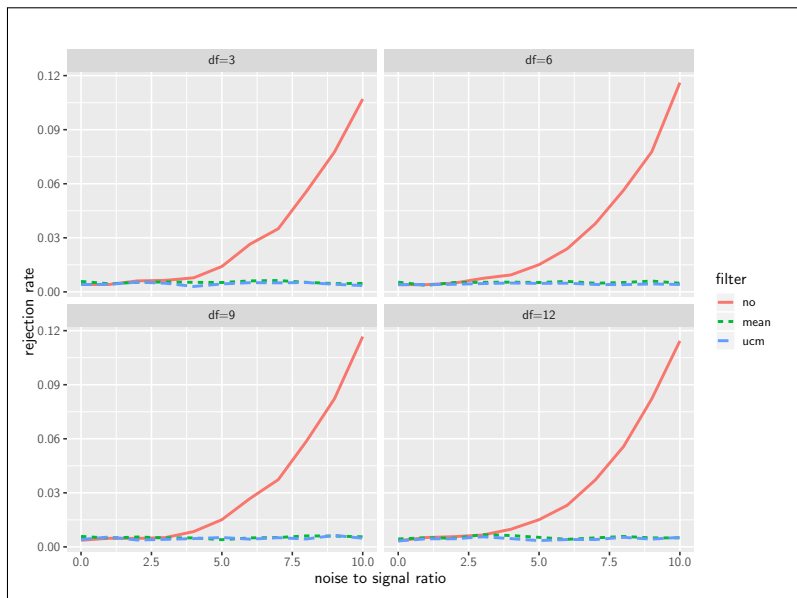
# Rejection rates for $H_0 : r = 1$ when $r = 2$



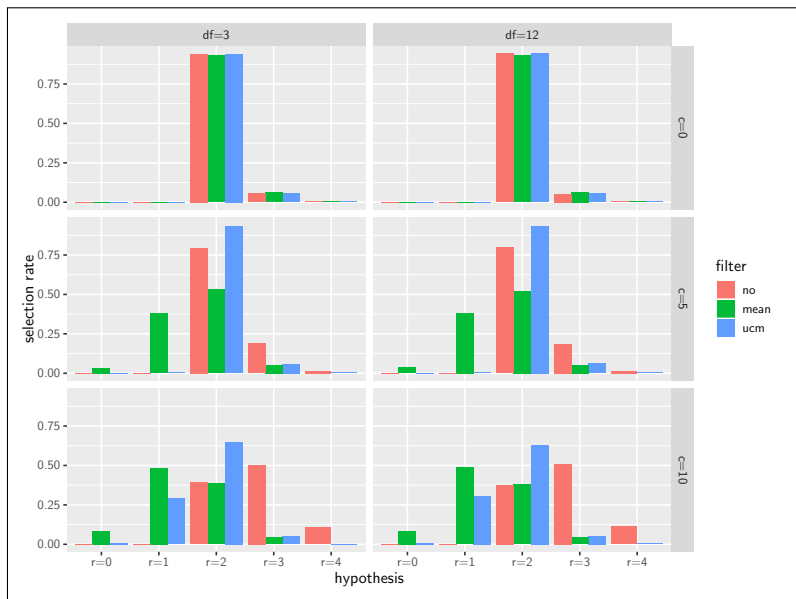
# Rejection rates for $H_0 : r = 2$ when $r = 2$



# Rejection rates for $H_0 : r = 3$ when $r = 2$



# Selection rates when $r = 2$



# Conclusions

- ADF and Johansen tests fail when data are very noisy: bias towards **more stationarity**.
- Filtering partially solves, but if there is too much noise bias towards **more integration**.
- Filtering based on **UCM smoothing** seem to work the best.
- Filtering should become routine when looking at long-run features of electricity prices.
- We need to work out theoretical results for UCM-smoothed time series of trend.