Hedging the Crack Spread

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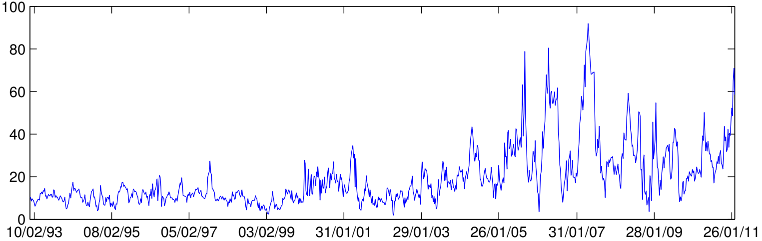
The Crack Spread

- $a : b : c$ crack spread - going long a barrels of crude oil, short b barrels of gasoline and short c barrels of heating oil



3:2:1 crack spread spot price in USD per bundle

- Futures contracts are used to hedge weekly spot crack spread positions

Literature Review

- Calculating the hedge ratios requires the estimation of the covariance matrix of the hedged portfolio
- Most works support the GARCH method:
    - Cecchetti et al. (1988), Baillie and Myers (1991), Kroner and Sultan (1993), Gagnon et al. (1998), Haigh and Holt (2000), Haigh and Holt (2002), Lee and Yorder (2007), Lien (2008), Lee (2009), Lee (2010), Chang et al. (2011) and Ji and Fan (2011)

Gaps in the Literature

- There are a number of ways to achieve time-varying variances and covariances other than GARCH including EWMA and Rolling OLS
- Statistical significance between model performances is seldom analysed
- Transaction costs are usually ignored or based on outdated rules

Our Contributions

- We provide an extensive performance comparison of different covariance estimators in hedging of the crack spread
- We examine the performance of each model relatively to the naïve model and test for statistical significance in the differences between each pair
- We account for the newly established rules from the NYMEX when calculating transaction and margin requirements

Notation

- Commodities
  - c - crude oil
  - g - gasoline
  - h - heating Oil
  - z - crack spread
- Spot, Futures
  - S - spot
  - F - futures
- e.g. S_t^c refers to the crude oil spot price at time t

Minimum-Variance Hedge Ratios

- For a hedged portfolio profit and loss (P&L),

$$\Delta\Pi_t = \Delta S_t^z - a\beta^c \Delta F_t^c + b\beta^g \Delta F_t^g + c\beta^h \Delta F_t^h \ ,$$

FOC:

$$\boldsymbol{\beta} = -V[\Delta \mathbf{F}_t]^{-1} COV[\Delta \mathbf{F}_t, \Delta S_t^z] \ ,$$

- This is analogous to performing a multiple regression of ΔS_t^z on the three separate futures P&L

Hedging Models

- Minimum-variance hedging models are catagorised by the different regression configurations of spot and futures P&Ls:
  1. Single-equation, multiple-variable
  $$\Delta S_t^z = \beta_0 + a\beta^c \Delta F_t^c - b\beta^g \Delta F_t^g - c\beta^h \Delta F_t^h + \epsilon_t \ ,$$
  2. Multiple-equation, single-variable
  $$\begin{bmatrix} \Delta S_t^c \\ \Delta S_t^g \\ \Delta S_t^h \end{bmatrix} = \begin{bmatrix} \beta_0^c + \beta^c \Delta F_t^c + \epsilon_t^c \\ \beta_0^g + \beta^g \Delta F_t^g + \epsilon_t^g \\ \beta_0^h + \beta^h \Delta F_t^h + \epsilon_t^h \end{bmatrix} \ ,$$
  3. Single-equation, single-variable
  $$\Delta S_t^z = \beta_0 + \beta^z \Delta F_t^z + \epsilon_t \ .$$
- For naïve model, $\beta^z = 1$

Estimation Methods I

- Rolling moving average (RMA) or Ordinary Least Squares (OLS)

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_1,t} = \frac{1}{n-1}\sum_{i=0}^{n}(\Delta Y_{1,t-i} - \Delta\bar{Y}_{1,t})^2 \ ,$$

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_2,t} = \frac{1}{n-1}\sum_{i=0}^{n}(\Delta Y_{1,t-i} - \Delta\bar{Y}_{1t})(\Delta Y_{2,t-i} - \Delta\bar{Y}_{2,t}) \ ,$$

- Exponentially weighted moving average (EWMA)

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_1,t} = (1-\lambda)\Delta Y_{1,t-1}^2 + \lambda\hat{\sigma}_{\Delta Y_1,t-1}^2 \ ,$$

$$\hat{\sigma}_{\Delta Y_1 \Delta Y_2,t} = (1-\lambda)\Delta Y_{1,t-1}\Delta Y_{2,t-1} + \lambda\hat{\sigma}_{\Delta Y_1 \Delta Y_2,t-1} \ .$$

where $\hat{\sigma}_{ij,t}$ denotes the estimate of the covariance between i and j at time t, n is the number of observations in the rolling window and $\lambda \in [0,1]$

Estimation Methods II

- AGARCH, aka asymmetric BEKK

$$\begin{aligned} \hat{\mathbf{H}}_t &= \mathbf{A}'\mathbf{A} + (\mathbf{B}'\boldsymbol{\Delta}\mathbf{Y}_{t-1})(\mathbf{B}'\boldsymbol{\Delta}\mathbf{Y}_{t-1})' + \\ &\quad + \mathbf{C}'\hat{\mathbf{H}}_{t-1}\mathbf{C} + (\mathbf{D}'\boldsymbol{\Delta}\mathbf{Y}^*_{t-1})(\mathbf{D}'\boldsymbol{\Delta}\mathbf{Y}^*_{t-1})' \ , \end{aligned}$$

  where $\hat{\mathbf{H}}_t$ is the covariance matrix estimate at time t and the parameters $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are found by maximising the log-likelihood function

$$\ln L(\theta) = -\frac{1}{2} \sum_{t=1}^{n} (\ln(|\mathbf{H}_t|) + \boldsymbol{\Delta}\mathbf{Y}_t'\mathbf{H}_t^{-1}\boldsymbol{\Delta}\mathbf{Y}_t) \ ,$$

- For symmetric GARCH, take $\mathbf{D}$ as a matrix of zeros with the same dimensions as the covariance matrix

Model Notation

- The hedging models are denoted by M_{ij}
  - M: estimation method
  - i: number of equations and
  - j: number of variables

- Seven hedging models are analysed:
  - Naïve
  - OLS_{31}
  - OLS_{13}
  - OLS_{11}
  - EWMA_{11}
  - GARCH_{11}
  - AGARCH_{11}

Analysis Outline

- Covariance matrix estimates at time t are used to calculate the hedge ratios for rebalancing the portfolio held from time t to $t+1$

- OLS hedge ratios and the GARCH parameters are calculated using a rolling window of 260 weeks

- EWMA parameter $\lambda = 0.99$

Margin and Transaction costs

- Bid-ask Spreads from Dunis et al. (2008):
  - Crude oil - 1 bps
  - Gasoline - 10 bps
  - Heating oil - 12 bps
- All bid-ask spreads are assumed to be constant over time
- The refinery is assumed to raise debt in financing the initial margins. Moodys AA index is used as proxy for cost of debt
- The 3 month US T-bill rate is used as a proxy for the risk-free rate
- The initial margin is assumed to remain constant at 10 USD per 3:2:1 bundle

Hedging Effectiveness

- Ederington Effectiveness (EE) is used as the performance metric:

$$\text{EE} = \frac{\sigma_u^2 - \sigma_h^2}{\sigma_u^2} ,$$

  where σ_u^2 and σ_h^2 denote unhedged and hedged portfolio variances respectively and are calculated

  1. using the entire hedged portfolio sample
  2. using the EWMA method

- The F-test for equal variance is applied to statistically distinguish between the hedged portfolio variances derived from each model

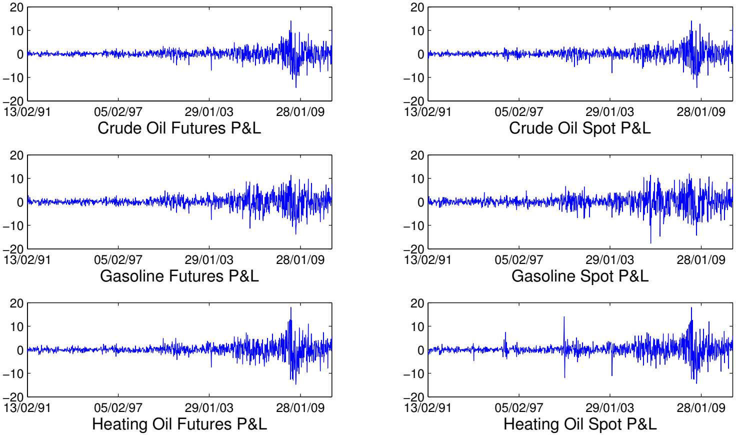
Spot Data

- Range: 30/12/1992 to 01/03/2011
- Frequency: weekly
- Source: Platts
- Varies greatly from different sources
- For crude oil, refineries buy at the price of the *benchmark* oil, i.e. WTI light sweet crude plus a differential
- Spot prices are prone to sudden changes in demand and supply which causes spikes in the data on a regular basis

Futures Data

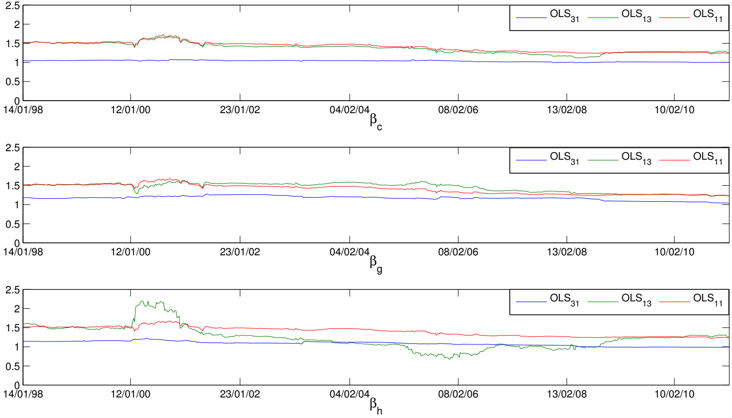
- Range: 30/12/1992 to 01/03/2011
- Frequency: weekly
- How to compute a continuous futures series?
  - Standard method: roll-over series
  - Constant maturity method
- Problems with the roll-over method:
  - Samuelson effect: futures price volatility increases with decreasing time to maturity
  - Causes regression between spot and non-constant maturity futures to become biased
- Constant-maturity futures method preferred to roll-over method although these require rebalancing every period

Futures and Spot P&L evolution

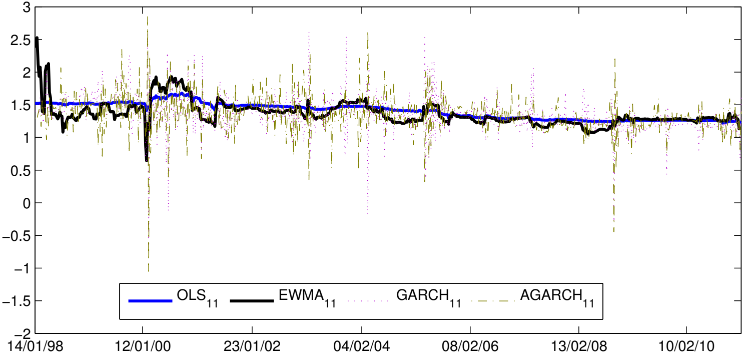


All P&Ls quoted in USD per Barrel

Hedge Ratios: OLS



Hedge Ratios: EWMA, GARCH and AGARCH



Nominal Transaction and Margin Costs

	In-sample		Out-of-sample	
Naïve	0.00	(0.10)	0.00	(0.10)
OLS_{31}	0.00	(0.30)	0.90	(0.80)
OLS_{13}	0.00	(0.10)	1.00	(1.00)
OLS_{11}	0.90	(0.40)	1.10	(0.60)
$EWMA_{11}$	1.40	(1.10)	1.40	(1.10)
$GARCH_{11}$	2.40	(2.70)	6.20	(7.30)
$AGARCH_{11}$	3.60	(4.00)	6.20	(7.00)

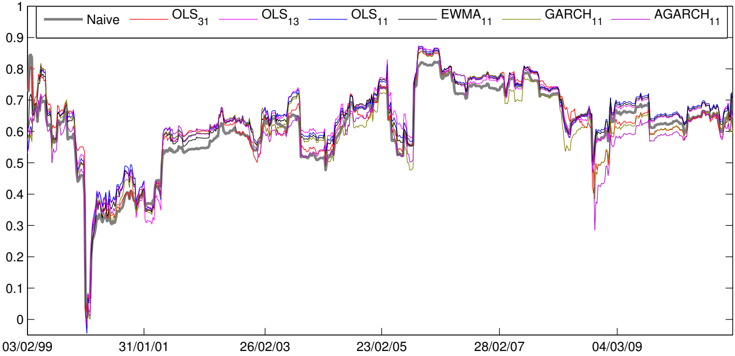
- Average transaction and margins costs in USD-cents per spot bundle, with standard deviations in parentheses

Ederington Effectiveness

	In-sample	Out-of-sample
Naïve	66.68%	66.68%
OLS_{31}	67.48%	67.32%
OLS_{13}	69.38%	69.15%
OLS_{11}	69.86%	69.70%
$EWMA_{11}$	69.09%	69.09%
$GARCH_{11}$	66.85%	66.31%
$AGARCH_{11}$	65.61%	66.43%

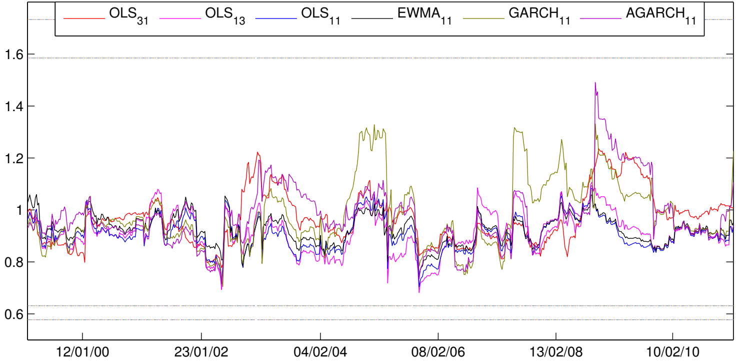
- Out-of-sample unconditional EE of each model including transaction and margin costs

Time-Varying Ederington Effectiveness



- Out-of-sample EE estimated using the EWMA method with $\lambda = 0.99$
- Initial variances estimated using the first 52 data points of hedged portfolio P&L

Rolling Moving Average F-Statistic



- Test for equal variance relative the naïve hedged portfolio variance
- F statistics calculated using a rolling window of 52 weeks
- Outer horizontal lines indicate the critical values at 90% and 95% significance respectively

Effect of Transaction and Margin costs

	In-sample	Out-of-sample
Naïve	0.017%	0.017%
OLS_{31}	0.016%	0.016%
OLS_{13}	0.016%	0.018%
OLS_{11}	0.023%	0.025%
$EWMA_{11}$	0.033%	0.033%
$GARCH_{11}$	0.056%	0.203%
$AGARCH_{11}$	0.087%	0.204%

- Reduction in EE of each model from adding transaction and margins costs
- Sample period 30/12/1992 - 01/03/2011

Conclusions

- All models reduce the variance of the unhedged portfolio by approximately 65-70% except during periods with abnormal market conditions
- All models are statistically indistiguishable from the naïve model at 10% significance level
- Minimum-variance hedge ratios require a significant amount of transaction and margin costs to implement, especially those calculated using the GARCH methods
- The investor should favour the naïve over other minimum-variance hedges