Structural models of the Italian electricity supply stack

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Outline of the talk

- Background and motivation
  - Curvature, spikes, and leverage effects
- Supply stack models
  - Power function (direct fit)
  - Logistic quantile function (indirect fit)
- Empirical application
  - Italian Power Exchange (Ipex) bid data
- Curvature of the supply stack
  - Convex or concave?
  - How does curvature respond to demand fluctuations?
- Concluding remarks

Well known phenomena in electricity markets empirics

- Spikes
  - Sharp and short-lived price excursions
  - More likely as demand approaches system capacity / causes congestion
- Inverse leverage effect
  - Electricity price volatility increases more after positive shocks than after negative shocks
  - Tested through EGARCH and TARCH models
  - Hadsell et al. (2004), Bowden and Payne (2008): confirm
  - Petrella and Sapio (2012): mixed evidence controlling for institutional change

Spikes, leverage, and the supply stack

- Convexity in the supply stack
  - $\Rightarrow$ Spikes (e.g. Simonsen 2004)
  - $\Rightarrow$ Inverse leverage effect (Kanamura)
- Convexity is a realistic assumption
  - Satisfying increasing electricity demand requires increasingly expensive energy sources
  - The market-wide marginal cost of electricity is convex
  - Strategic reasons to ”inflate” bids more on the less efficient units (Ausubel and Cramton 1996)

Spikes, leverage, and the supply stack

- However, the supply stack curvature may not be constant with respect to demand
  - Demand spanning a given highly non-linear supply stack
  - Supply decisions are revised over time to exploit the profit opportunities triggered by increasing demand
- The availability of bid-level data allows to investigate the shape of the supply stack directly and detect how supply interacts with demand...
- ...well beyond the simple crossing of the market curves

Structural models of the electricity supply stack

- Definitions
  - $B_{(i)}$: i-th price bid in the merit order
  - $Q_{(i)}$: quantity offered through the i-th bid in the merit order
  - $Q_i \equiv \sum_{j=1}^{i} Q_{(j)}$: quantity offered through the best i bids in the merit order
  - $\tilde{Q}_i \equiv \frac{Q_i - \underline{Q}}{\overline{Q} - \underline{Q}}$ - trimming the lower and upper tails

Structural models of the electricity supply stack

- Power

$$B_{(i)} = \alpha Q_i^{\beta}$$

with $\alpha, \beta > 0$
  - Convex if $\beta > 1$
  - Mount (2000) - with a shift
- Logistic quantile function

$$B_{(i)} = m + s \ln \frac{\tilde{Q}_i}{1 - \tilde{Q}_i}$$

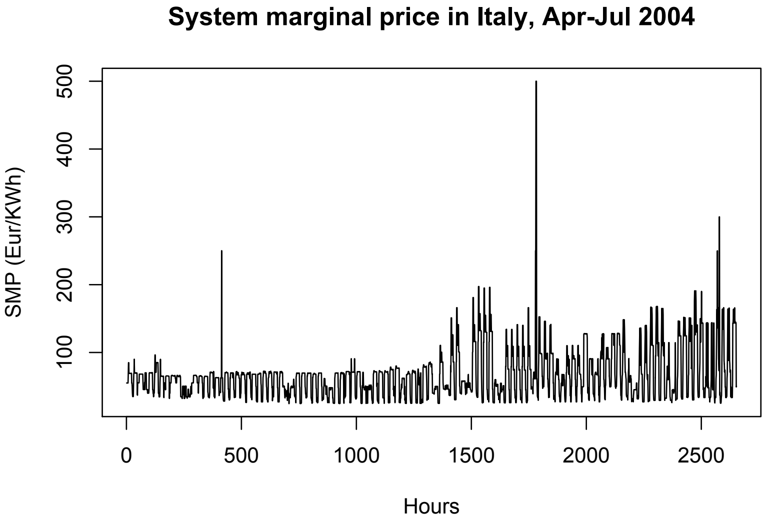
  - Convex (concave) for all prices above (below) the median bid
  - Coulon and Howison (2010)

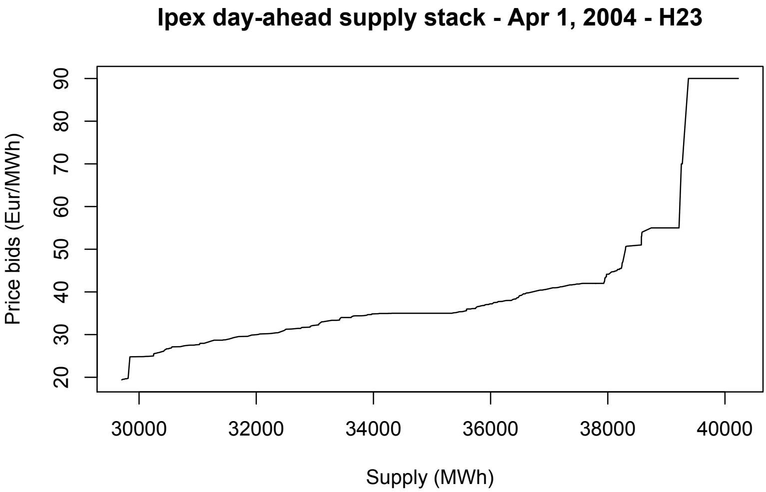
Alternative models not explored here

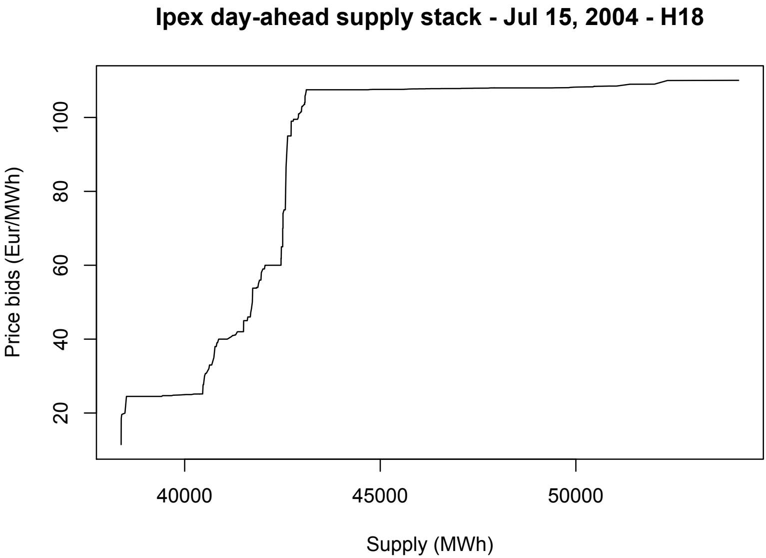
- Exponential
  - Convex for all parameter values - convexity cannot be falsified
  - Skantze et al. 2000, Lucia and Schwartz 2002, Burger et al. 2004, Villaplana 2006, Pirrong and Jermakyan 2008, Cartea and Villaplana 2008
- Hockey-stick, Inverse Box-Cox
  - Soon to be estimated - more computationally expensive
  - (Kanamura and Ohashi 2004, 2008) (Barlow 2002, Kanamura 2009)

The dataset

- Individual price-quantity offers on the day-ahead Italian electricity market
- April-July 2004 (122 days, 24 market sessions per day)
  - Highest market concentration / State-owned dominant player: Enel
  - Uniform market architecture within that period
  - Remove zero-price offers (contracts, hydro, other renewables) and offers above the 0.9 quantile of the price bid distribution

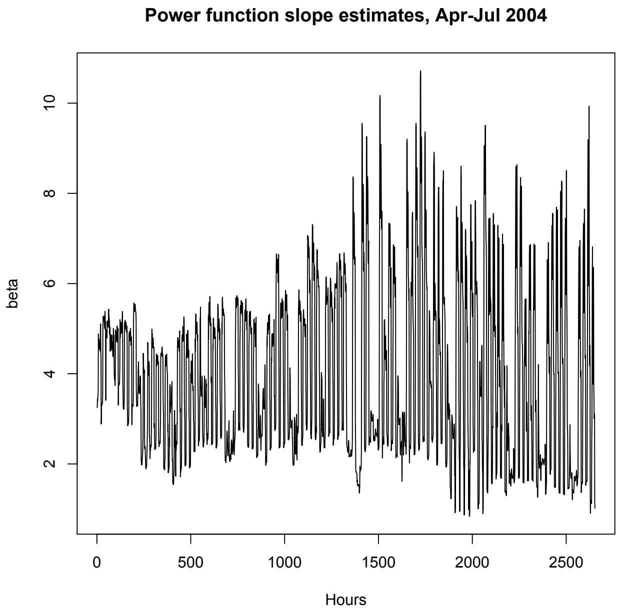

System marginal price in Italy, Apr-Jul 2004
SMP (Eur/KWh)
100
200
300
400
500
0
500
1000
1500
2000
2500
Hours

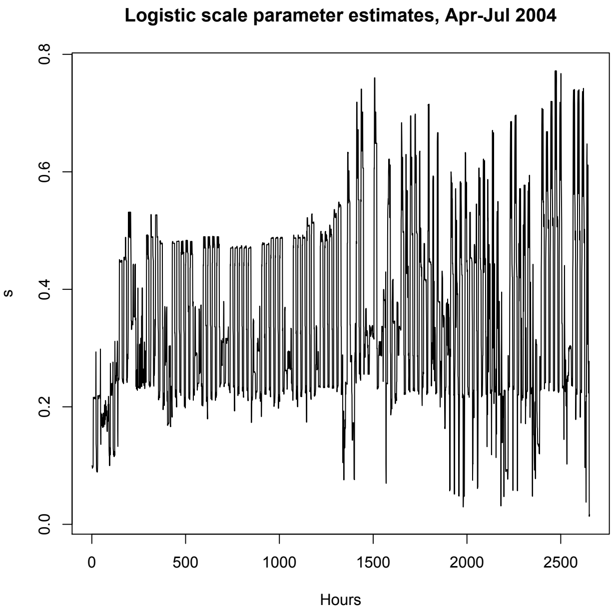

Ipex day-ahead supply stack - Apr 1, 2004 - H23
Price bids (Eur/MWh)
20
30
40
50
60
70
80
90
30000
32000
34000
36000
38000
40000
Supply (MWh)


Ipex day-ahead supply stack - Jul 15, 2004 - H18
Price bids (Eur/MWh)
20
40
60
80
100
40000
45000
50000
Supply (MWh)

Fitting the supply stack

- Direct fit
  - Sort offers by increasing prices and fit the stack
  - 435 observations per hour on average (min: 182, max: 731)
  - Fit the supply stack with a power function $\rightarrow \hat{\alpha}$, $\hat{\beta}$
- Distribution fit
  - Treat each MWh as a different observation (as Coulon and Howison 2010)
  - 15,597,808 observations per hour on average (min: 2,281,224; max: 33,489,168)
  - Fit the empirical density function (using a logistic distribution) $\rightarrow \hat{m}$, $\hat{s}$


Power function slope estimates, Apr-Jul 2004
beta
2
4
6
8
10
0
500
1000
1500
2000
2500
Hours


Logistic scale parameter estimates, Apr-Jul 2004
s
0.0
0.2
0.4
0.6
0.8
0
500
1000
1500
2000
2500
Hours

Convex or concave?

- Evaluate second derivatives at the level of demand D
- Power function curvature

$$C_{pwr} \equiv \frac{d^2 B(D)}{dQ^2} = \hat{\alpha}\hat{\beta}(\hat{\beta} - 1)D^{\hat{\beta}-2}$$

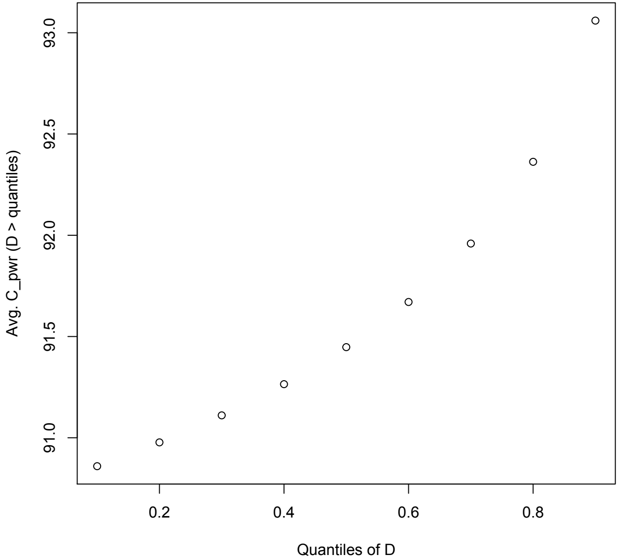
- Logistic quantile function curvature

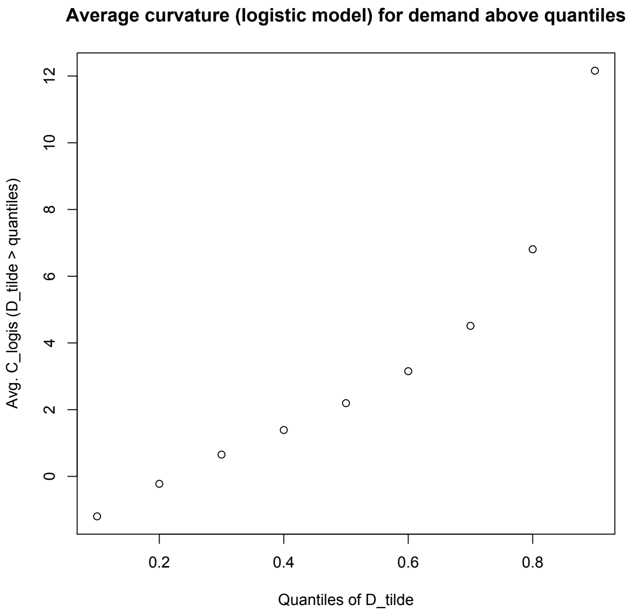
$$C_{logis} \equiv \frac{d^2 B(\tilde{D})}{dQ^2} = \hat{s}\frac{2\tilde{D} - 1}{\tilde{D}^2(1 - \tilde{D})^2}$$

where

- $\hat{\alpha}$, $\hat{\beta}$: parameter estimates (direct fit)
- $\hat{s}$: parameter estimate (ML density fit)
- $\tilde{D} \equiv \frac{D-\underline{Q}}{\overline{Q}-\underline{Q}}$

…odel) for demand above quantiles




Average curvature (logistic model) for demand above quantiles
Avg. C_logis (D_tilde > quantiles)
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
Quantiles of D_tilde

How does curvature respond to demand fluctuations?

- Curvature depends on demand via two channels
  - Demand spanning a given non-linear supply stack
  - Supply stack dynamics induced by expected demand
- Decomposing the demand-elasticity of curvature: **supply non-linearity** + **supply dynamics**

$$\frac{d\log|C_{pwr}|}{d\log D} = \frac{d\log\hat{\alpha}}{d\log D} + \frac{d\log\hat{\beta}}{d\log D}\left(1 + \frac{\hat{\beta}}{|\hat{\beta}-1|} + \hat{\beta}\log D\right) + (\hat{\beta}-2)$$

$$\frac{d\log|C_{logis}|}{d\log\tilde{D}} = \frac{d\log\hat{s}}{d\log\tilde{D}} + 2\left(\frac{\tilde{D}}{|2\tilde{D}-1|} + \frac{2\tilde{D}-1}{1-\tilde{D}}\right)$$

Supply dynamics - the power model

Table: Log-linear regression of $\hat{\beta}$ on D

	coeff. (s.e.)	coeff (s.e.)	coeff. (s.e.)
const	-15.995 (0.363)	3.477 (0.833)	3.347 (0.833)
log D	1.653 (0.035)	1.883 (0.032)	1.891 (0.032)
log fuel index		-6.449 (0.254)	-6.453 (0.254)
n. congested lines			0.014 (0.005)
n. obs.	2653	2653	2653
R^2	0.461	0.567	0.568

Supply dynamics - the quantile logistic model

Table: Log-linear regression of $\hat{s}$ on $\tilde{D}$

	coeff. (s.e.)	coeff (s.e.)	coeff. (s.e.)
const	-0.785	-11.418	-11.539
	(0.018)	(1.145)	(1.147)
log $\tilde{D}$	0.299	0.342	0.344
	(0.013)	(0.014)	(0.014)
log fuel index		3.150	3.175
		(0.339)	(0.339)
n. congested lines			0.009
			(0.006)
n. obs.	2653	2653	2653
R^2	0.160	0.186	0.187

Decompositions of the demand-elasticity of curvature

Table: Demand-elasticities of curvature: components across demand quantiles

Quantiles	$min(D)$	$median(D)$	$max(D)$
Power model			
Supply non-linearity	0.885	-0.050	5.449
Supply dynamics	56.304	40.589	137.081
Quantile logistic model			
Supply non-linearity	-1.986	1.212	33.727
Supply dynamics	0.344	0.344	0.344

Curvature is more a 'supply non-linearity' issue (and less due to 'supply dynamics') when demand is high than when demand is low

Conclusion

- Summary of results
  - The Italian supply stack changes even on a short horizon $\Leftarrow$ noise + deliberate behaviors
  - The power function fit confirms convexity, while the logistic fit sheds light on concave parts of the supply stack
  - When demand is high, curvature is mainly due to non-linearity of the supply stack, and responds less to its change over time
- To do's:
  - Extending the dataset to cover more recent periods
  - Fitting further non-linear models
  - Perform goodness-of-fit comparisons (or develop ways to do so)
  - More suggestions by the audience...